LENAPE VALLEY REGIONAL HIGH SCHOOL

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Summer Reading
AP Physics I

The goal of this summer assignment is to provide you with a foundation that we will build upon on the first day of class. This reading will provide you with an overview of Physics and the Mathematical concepts we will use throughout the year. I have designed the assignment so that you do not need your physics text. There are two PDFs that you will utilize to complete this assignment. The first is a copy of the first chapter of your textbook. The second is a list of multiple choice questions you will complete as you read the chapter. You can work on this by the pool as long as you have a wireless internet connection or you can print out all the pages. I will be collecting the multiple choice questions on the first day of class.

Try to pace yourself during the summer so that you are not rushed at the end. All the questions are in order and labeled according to the section of the book they correspond to. Should you have any questions please email me at Pdrury@lvhs.org

I have also attached the course description for your information.

Prerequisite: Successful completion of Honors Chemistry, Algebra 2 Honors, and Biology Honors with a combined average of "B" or higher. Students in CP Biology, CP Chemistry and CP level math with grades of (A-) or higher can also experience success in this course.

AP Physics 1: is the equivalent to a first-semester college course in algebra-based physics. The course covers Newtonian mechanics (including rotational dynamics and angular momentum); work, energy, and power; and mechanical waves and sound and simple circuits. Through inquiry based learning, students will develop scientific thinking and reasoning skills. It is expected that students will complete 5 -10 hours a week of independent study outside of class. All students enrolled in this course are expected to take the Advanced Placement Examination.

Thanks and have a great summer ☺
Mr. Drury

CHAPTER

1

The animation techniques and special effects used in the film *Avatar* rely on computers and mathematical concepts such as trigonometry and vectors. Such mathematical concepts will also be useful throughout this book in our discussion of physics.

Introduction and Mathematical Concepts

1.1 The Nature of Physics

The science of physics has developed out of the efforts of men and women to explain our physical environment. These efforts have been so successful that the laws of physics now encompass a remarkable variety of phenomena, including planetary orbits, radio and TV waves, magnetism, and lasers, to name just a few.

The exciting feature of physics is its capacity for predicting how nature will behave in one situation on the basis of experimental data obtained in another situation. Such predictions place physics at the heart of modern technology and, therefore, can have a tremendous impact on our lives. Rocketry and the development of space travel have their roots firmly planted in the physical laws of Galileo Galilei (1564–1642) and Isaac Newton (1642–1727). The transportation industry relies heavily on physics in the development of engines and the design of aerodynamic vehicles. Entire electronics and computer industries owe their existence to the invention of the transistor, which grew directly out of the laws of physics that describe the electrical behavior of solids. The telecommunications industry depends extensively on electromagnetic waves, whose existence was predicted by James Clerk Maxwell (1831–1879) in his theory of electricity and magnetism. The medical profession uses X-ray, ultrasonic, and magnetic resonance methods for obtaining images of the interior of the human body, and physics lies at the core of all these. Perhaps the most widespread impact in modern technology is that due to the laser. Fields ranging from space exploration to medicine benefit from this incredible device, which is a direct application of the principles of atomic physics.

Because physics is so fundamental, it is a required course for students in a wide range of major areas. We welcome you to the study of this fascinating topic. You will learn how to see the world through the "eyes" of physics and to reason as a physicist does. In the process, you will learn how to apply physics principles to a wide range of problems. We hope that you will come to recognize that physics has important things to say about your environment.

1.2 Units

Physics experiments involve the measurement of a variety of quantities, and a great deal of effort goes into making these measurements as accurate and reproducible as possible. The first step toward ensuring accuracy and reproducibility is defining the units in which the measurements are made.

Figure 1.1 The standard platinum–iridium meter bar. (Courtesy Bureau International des Poids et Mesures, France)

Table 1.1 Units of Measurement

	System		
	SI	CGS	BE
Length	Meter (m)	Centimeter (cm)	Foot (ft)
Mass	Kilogram (kg)	Gram (g)	Slug (sl)
Time	Second (s)	Second (s)	Second (s)

In this text, we emphasize the system of units known as ***SI units,*** which stands for the French phrase "Le **S**ystème **I**nternational d'Unités." By international agreement, this system employs the ***meter*** (m) as the unit of length, the ***kilogram*** (kg) as the unit of mass, and the ***second*** (s) as the unit of time. Two other systems of units are also in use, however. The CGS system utilizes the centimeter (cm), the gram (g), and the second for length, mass, and time, respectively, and the BE or British Engineering system (the gravitational version) uses the foot (ft), the slug (sl), and the second. Table 1.1 summarizes the units used for length, mass, and time in the three systems.

Originally, the meter was defined in terms of the distance measured along the earth's surface between the north pole and the equator. Eventually, a more accurate measurement standard was needed, and by international agreement the meter became the distance between two marks on a bar of platinum–iridium alloy (see Figure 1.1) kept at a temperature of 0 °C. Today, to meet further demands for increased accuracy, the meter is defined as the distance that light travels in a vacuum in a time of 1/299 792 458 second. This definition arises because the speed of light is a universal constant that is defined to be 299 792 458 m/s.

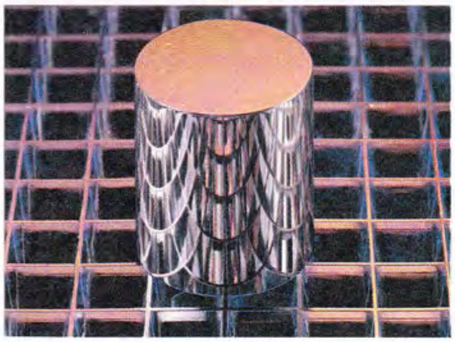


Figure 1.2 The standard platinum–iridium kilogram is kept at the International Bureau of Weights and Measures in Sèvres, France. This copy of it was assigned to the United States in 1889 and is housed at the National Institute of Standards and Technology. (Copyright Robert Rathe, National Institute of Standards and Technology)

The definition of a kilogram as a unit of mass has also undergone changes over the years. As Chapter 4 discusses, the mass of an object indicates the tendency of the object to continue in motion with a constant velocity. Originally, the kilogram was expressed in terms of a specific amount of water. Today, one kilogram is defined to be the mass of a standard cylinder of platinum–iridium alloy, like the one in Figure 1.2.

As with the units for length and mass, the present definition of the second as a unit of time is different from the original definition. Originally, the second was defined according to the average time for the earth to rotate once about its axis, one day being set equal to 86 400 seconds. The earth's rotational motion was chosen because it is naturally repetitive, occurring over and over again. Today, we still use a naturally occurring repetitive phenomenon to define the second, but of a very different kind. We use the electromagnetic waves emitted by cesium-133 atoms in an atomic clock like that in Figure 1.3. One second is defined as the time needed for 9 192 631 770 wave cycles to occur.*

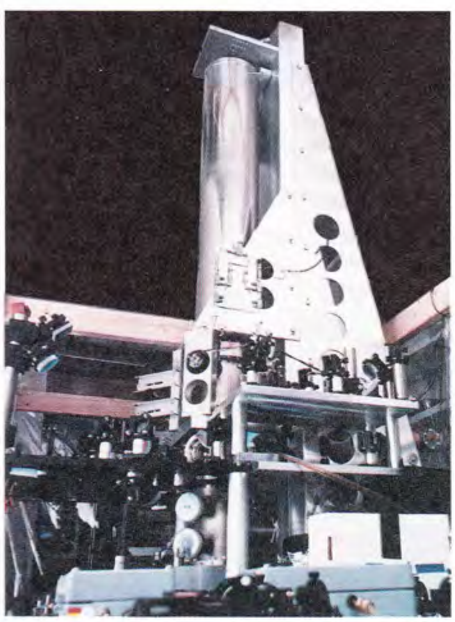


Figure 1.3 This atomic clock, the NIST-F1, keeps time with an uncertainty of about one second in sixty million years. (© Geoffrey Wheeler)

The units for length, mass, and time, along with a few other units that will arise later, are regarded as ***base*** SI units. The word "base" refers to the fact that these units are used along with various laws to define additional units for other important physical quantities, such as force and energy. The units for such other physical quantities are referred to as ***derived*** units, since they are combinations of the base units. Derived units will be introduced from time to time, as they arise naturally along with the related physical laws.

The value of a quantity in terms of base or derived units is sometimes a very large or very small number. In such cases, it is convenient to introduce larger or smaller units that are related to the normal units by multiples of ten. Table 1.2 summarizes the prefixes that are used to denote multiples of ten. For example, 1000 or 10^3 meters are referred to as 1 kilometer (km), and 0.001 or 10^{-3} meter is called 1 millimeter (mm). Similarly, 1000 grams and 0.001 gram are referred to as 1 kilogram (kg) and 1 milligram (mg), respectively. Appendix A contains a discussion of scientific notation and powers of ten, such as 10^3 and 10^{-3}.

*See Chapter 16 for a discussion of waves in general and Chapter 24 for a discussion of electromagnetic waves in particular.

1.3 The Role of Units in Problem Solving

The Conversion of Units

Since any quantity, such as length, can be measured in several different units, it is important to know how to convert from one unit to another. For instance, the foot can be used to express the distance between the two marks on the standard platinum–iridium meter bar. There are 3.281 feet in one meter, and this number can be used to convert from meters to feet, as the following example demonstrates.

Example 1 The World's Highest Waterfall

The highest waterfall in the world is Angel Falls in Venezuela, with a total drop of 979.0 m (see Figure 1.4). Express this drop in feet.

Reasoning When converting between units, we write down the units explicitly in the calculations and treat them like any algebraic quantity. In particular, we will take advantage of the following algebraic fact: Multiplying or dividing an equation by a factor of 1 does not alter an equation.

Solution Since 3.281 feet = 1 meter, it follows that (3.281 feet)/(1 meter) = 1. Using this factor of 1 to multiply the equation "Length = 979.0 meters," we find that

$$\text{Length} = (979.0\text{ m})(1) = (979.0\ \cancel{\text{meters}})\left(\frac{3.281\text{ feet}}{1\ \cancel{\text{meter}}}\right) = \boxed{3212\text{ feet}}$$

The colored lines emphasize that the units of meters behave like any algebraic quantity and cancel when the multiplication is performed, leaving only the desired unit of feet to describe the answer. In this regard, note that 3.281 feet = 1 meter also implies that (1 meter)/(3.281 feet) = 1. However, we chose not to multiply by a factor of 1 in this form, because the units of meters would not have canceled.

A calculator gives the answer as 3212.099 feet. Standard procedures for significant figures, however, indicate that the answer should be rounded off to four significant figures, since the value of 979.0 meters is accurate to only four significant figures. In this regard, the "1 meter" in the denominator does not limit the significant figures of the answer, because this number is precisely one meter by definition of the conversion factor. Appendix B contains a review of significant figures.

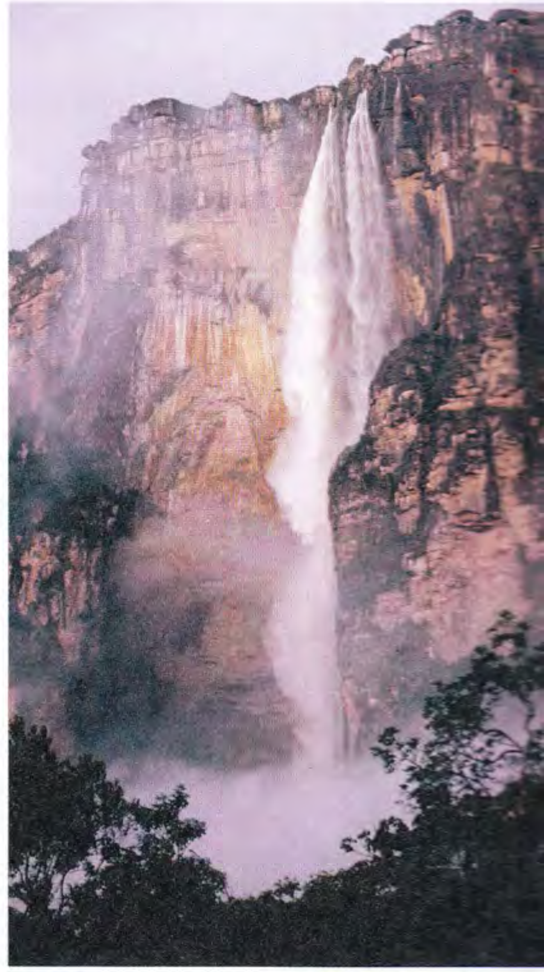


Figure 1.4 Angel Falls in Venezuela is the highest waterfall in the world. (© Andoni Canela/age fotostock)

Problem-Solving Insight. ***In any conversion, if the units do not combine algebraically to give the desired result, the conversion has not been carried out properly.*** With this in mind, the next example stresses the importance of writing down the units and illustrates a typical situation in which several conversions are required.

Example 2 Interstate Speed Limit

Express the speed limit of 65 miles/hour in terms of meters/second.

Reasoning As in Example 1, it is important to write down the units explicitly in the calculations and treat them like any algebraic quantity. Here, we take advantage of two well-known relationships—namely, 5280 feet = 1 mile and 3600 seconds = 1 hour. As a result, (5280 feet)/(1 mile) = 1 and (3600 seconds)/(1 hour) = 1. In our solution we will use the fact that multiplying and dividing by these factors of unity does not alter an equation.

Solution Multiplying and dividing by factors of unity, we find the speed limit in feet per second as shown below:

$$\text{Speed} = \left(65\ \frac{\text{miles}}{\text{hour}}\right)(1)(1) = \left(65\ \frac{\cancel{\text{miles}}}{\cancel{\text{hour}}}\right)\left(\frac{5280\text{ feet}}{1\ \cancel{\text{mile}}}\right)\left(\frac{1\ \cancel{\text{hour}}}{3600\text{ seconds}}\right) = 95\ \frac{\text{feet}}{\text{second}}$$

To convert feet into meters, we use the fact that (1 meter)/(3.281 feet) = 1:

$$\text{Speed} = \left(95\ \frac{\text{feet}}{\text{second}}\right)(1) = \left(95\ \frac{\cancel{\text{feet}}}{\text{second}}\right)\left(\frac{1\text{ meter}}{3.281\ \cancel{\text{feet}}}\right) = \boxed{29\ \frac{\text{meters}}{\text{second}}}$$

Table 1.2 Standard Prefixes Used to Denote Multiples of Ten

Prefix	Symbol	Factor[a]
tera	T	10^{12}
giga[b]	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

[a]Appendix A contains a discussion of powers of ten and scientific notation.
[b]Pronounced jig'a.

In addition to their role in guiding the use of conversion factors, units serve a useful purpose in solving problems. They can provide an internal check to eliminate errors, if they are carried along during each step of a calculation and treated like any algebraic factor. In particular, remember that ***only quantities with the same units can be added or subtracted*** (**Problem-Solving Insight**). Thus, at one point in a calculation, if you find yourself adding 12 miles to 32 kilometers, stop and reconsider. Either miles must be converted into kilometers or kilometers must be converted into miles before the addition can be carried out.

A collection of useful conversion factors is given on the page facing the inside of the front cover. The reasoning strategy that we have followed in Examples 1 and 2 for converting between units is outlined as follows:

Reasoning Strategy Converting Between Units

1. In all calculations, write down the units explicitly.
2. Treat all units as algebraic quantities. In particular, when identical units are divided, they are eliminated algebraically.
3. Use the conversion factors located on the page facing the inside of the front cover. Be guided by the fact that multiplying or dividing an equation by a factor of 1 does not alter the equation. For instance, the conversion factor of 3.281 feet = 1 meter might be applied in the form (3.281 feet)/(1 meter) = 1. This factor of 1 would be used to multiply an equation such as "Length = 5.00 meters" in order to convert meters to feet.
4. Check to see that your calculations are correct by verifying that the units combine algebraically to give the desired unit for the answer. Only quantities with the same units can be added or subtracted.

Sometimes an equation is expressed in a way that requires specific units to be used for the variables in the equation. In such cases it is important to understand why only certain units can be used in the equation, as the following example illustrates.

Example 3 The Physics of the Body Mass Index

The body mass index (BMI) takes into account your mass in kilograms (kg) and your height in meters (m) and is defined as follows:

$$\text{BMI} = \frac{\text{Mass in kg}}{(\text{Height in m})^2}$$

However, the BMI is often computed using the weight* of a person in pounds (lb) and his or her height in inches (in.). Thus, the expression for the BMI incorporates these quantities, rather than the mass in kilograms and the height in meters. Starting with the definition above, determine the expression for the BMI that uses pounds and inches.

Reasoning We will begin with the BMI definition and work separately with the numerator and the denominator. We will determine the mass in kilograms that appears in the numerator from the weight in pounds by using the fact that 1 kg corresponds to 2.205 lb. Then, we will determine the height in meters that appears in the denominator from the height in inches with the aid of the facts that 1 m = 3.281 ft and 1 ft = 12 in. These conversion factors are located on the page facing the inside of the front cover of the text.

Solution Since 1 kg corresponds to 2.205 lb, the mass in kilograms can be determined from the weight in pounds in the following way:

$$\text{Mass in kg} = (\text{Weight in lb})\left(\frac{1\ \text{kg}}{2.205\ \text{lb}}\right)$$

Since 1 ft = 12 in. and 1 m = 3.281 ft, we have

$$\text{Height in m} = (\text{Height in in.})\left(\frac{1\ \cancel{\text{ft}}}{12\ \text{in.}}\right)\left(\frac{1\ \text{m}}{3.281\ \cancel{\text{ft}}}\right)$$

*Weight and mass are different concepts, and the relationship between them will be discussed in Section 4.7.

Substituting these results into the numerator and denominator of the BMI definition gives

$$\text{BMI} = \frac{\text{Mass in kg}}{(\text{Height in m})^2} = \frac{(\text{Weight in lb})\left(\dfrac{1\text{ kg}}{2.205\text{ lb}}\right)}{(\text{Height in in.})^2\left(\dfrac{1\text{ ft}}{12\text{ in.}}\right)^2\left(\dfrac{1\text{ m}}{3.281\text{ ft}}\right)^2}$$

$$= \left(\frac{1\text{ kg}}{2.205\text{ lb}}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2\left(\frac{3.281\text{ ft}}{1\text{ m}}\right)^2 \frac{(\text{Weight in lb})}{(\text{Height in in.})^2}$$

$$\boxed{\text{BMI} = \left(703.0\,\frac{\text{kg}\cdot\text{in.}^2}{\text{lb}\cdot\text{m}^2}\right)\frac{(\text{Weight in lb})}{(\text{Height in in.})^2}}$$

For example, if your weight and height are 180 lb and 71 in., your body mass index is 25 kg/m^2. The BMI can be used to assess approximately whether your weight is normal for your height (see Table 1.3).

Table 1.3 The Body Mass Index

BMI (kg/m^2)	Evaluation
Below 18.5	Underweight
18.5–24.9	Normal
25.0–29.9	Overweight
30.0–39.9	Obese
40 and above	Morbidly obese

Dimensional Analysis

We have seen that many quantities are denoted by specifying both a number and a unit. For example, the distance to the nearest telephone may be 8 meters, or the speed of a car might be 25 meters/second. Each quantity, according to its physical nature, requires a certain *type* of unit. Distance must be measured in a length unit such as meters, feet, or miles, and a time unit will not do. Likewise, the speed of an object must be specified as a length unit divided by a time unit. In physics, the term ***dimension*** is used to refer to the physical nature of a quantity and the type of unit used to specify it. Distance has the dimension of length, which is symbolized as [L], while speed has the dimensions of length [L] divided by time [T], or [L/T]. Many physical quantities can be expressed in terms of a combination of fundamental dimensions such as length [L], time [T], and mass [M]. Later on, we will encounter certain other quantities, such as temperature, which are also fundamental. A fundamental quantity like temperature cannot be expressed as a combination of the dimensions of length, time, mass, or any other fundamental dimension.

Dimensional analysis is used to check mathematical relations for the consistency of their dimensions. As an illustration, consider a car that starts from rest and accelerates to a speed v in a time t. Suppose we wish to calculate the distance x traveled by the car but are not sure whether the correct relation is $x = \frac{1}{2}vt^2$ or $x = \frac{1}{2}vt$. We can decide by checking the quantities on both sides of the equals sign to see whether they have the same dimensions. If the dimensions are not the same, the relation is incorrect. For $x = \frac{1}{2}vt^2$, we use the dimensions for distance [L], time [T], and speed [L/T] in the following way:

$$x = \tfrac{1}{2}vt^2$$

Dimensions $$[\text{L}] \stackrel{?}{=} \left[\frac{\text{L}}{\cancel{\text{T}}}\right][\text{T}]^{\cancel{2}} = [\text{L}][\text{T}]$$

Dimensions cancel just like algebraic quantities, and pure numerical factors like $\frac{1}{2}$ have no dimensions, so they can be ignored. The dimension on the left of the equals sign does not match those on the right, so the relation $x = \frac{1}{2}vt^2$ cannot be correct. On the other hand, applying dimensional analysis to $x = \frac{1}{2}vt$, we find that

Problem-Solving Insight. You can check for errors that may have arisen during algebraic manipulations by performing a dimensional analysis on the final expression.

$$x = \tfrac{1}{2}vt$$

Dimensions $$[\text{L}] \stackrel{?}{=} \left[\frac{\text{L}}{\cancel{\text{T}}}\right][\cancel{\text{T}}] = [\text{L}]$$

The dimension on the left of the equals sign matches that on the right, so this relation is dimensionally correct. If we know that one of our two choices is the right one, then $x = \frac{1}{2}vt$ is it. In the absence of such knowledge, however, dimensional analysis cannot identify the correct relation. It can only identify which choices *may be* correct, since it does not account for numerical factors like $\frac{1}{2}$ or for the manner in which an equation was derived from physics principles.

Check Your Understanding

(The answers are given at the end of the book.)

1. **(a)** Is it possible for two quantities to have the same dimensions but different units? **(b)** Is it possible for two quantities to have the same units but different dimensions?
2. You can always add two numbers that have the same units (such as 6 meters + 3 meters). Can you always add two numbers that have the same dimensions, such as two numbers that have the dimensions of length [L]?
3. The following table lists four variables, along with their units:

Variable	Units
x	Meters (m)
v	Meters per second (m/s)
t	Seconds (s)
a	Meters per second squared (m/s^2)

These variables appear in the following equations, along with a few numbers that have no units. In which of the equations are the units on the left side of the equals sign consistent with the units on the right side?

(a) $x = vt$ **(d)** $v = at + \frac{1}{2}at^3$
(b) $x = vt + \frac{1}{2}at^2$ **(e)** $v^3 = 2ax^2$
(c) $v = at$ **(f)** $t = \sqrt{\dfrac{2x}{a}}$

4. In the equation $y = c^n at^2$ you wish to determine the integer value (1, 2, etc.) of the exponent n. The dimensions of y, a, and t are known. It is also known that c has no dimensions. Can dimensional analysis be used to determine n?

1.4 Trigonometry

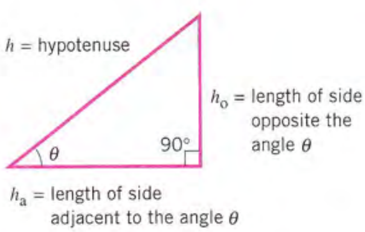


Figure 1.5 A right triangle.

Scientists use mathematics to help them describe how the physical universe works, and trigonometry is an important branch of mathematics. Three trigonometric functions are utilized throughout this text. They are the sine, the cosine, and the tangent of the angle θ (Greek theta), abbreviated as $\sin\theta$, $\cos\theta$, and $\tan\theta$, respectively. These functions are defined below in terms of the symbols given along with the right triangle in Figure 1.5.

Definition of Sin θ, Cos θ, and Tan θ

$$\sin\theta = \frac{h_o}{h} \tag{1.1}$$

$$\cos\theta = \frac{h_a}{h} \tag{1.2}$$

$$\tan\theta = \frac{h_o}{h_a} \tag{1.3}$$

h = length of the **hypotenuse** of a right triangle
h_o = length of the side **opposite** the angle θ
h_a = length of the side **adjacent** to the angle θ

The sine, cosine, and tangent of an angle are numbers without units, because each is the ratio of the lengths of two sides of a right triangle. Example 4 illustrates a typical application of Equation 1.3.

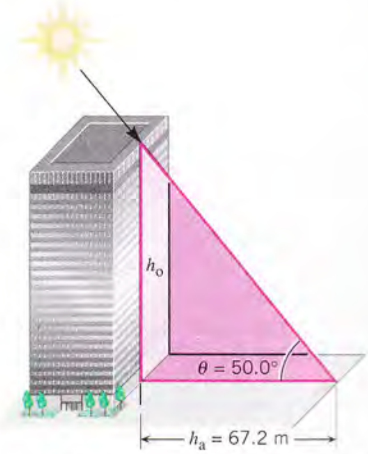


Figure 1.6 From a value for the angle θ and the length h_a of the shadow, the height h_o of the building can be found using trigonometry.

Example 4 Using Trigonometric Functions

On a sunny day, a tall building casts a shadow that is 67.2 m long. The angle between the sun's rays and the ground is $\theta = 50.0°$, as Figure 1.6 shows. Determine the height of the building.

Reasoning We want to find the height of the building. Therefore, we begin with the colored right triangle in Figure 1.6 and identify the height as the length h_o of the side opposite the angle θ. The length of the shadow is the length h_a of the side that is adjacent to the angle θ. The ratio of the length of the opposite side to the length of the adjacent side is the tangent of the angle θ, which can be used to find the height of the building.

Solution We use the tangent function in the following way, with $\theta = 50.0°$ and $h_a = 67.2$ m:

$$\tan\theta = \frac{h_o}{h_a} \tag{1.3}$$

$$h_o = h_a \tan\theta = (67.2\text{ m})(\tan 50.0°) = (67.2\text{ m})(1.19) = \boxed{80.0\text{ m}}$$

The value of tan 50.0° is found by using a calculator.

The sine, cosine, or tangent may be used in calculations such as that in Example 4, depending on which side of the triangle has a known value and which side is asked for. However, ***the choice of which side of the triangle to label h_o (opposite) and which to label h_a (adjacent) can be made only after the angle θ is identified.***

■ **Problem-Solving Insight.**

Often the values for two sides of the right triangle in Figure 1.5 are available, and the value of the angle θ is unknown. The concept of ***inverse trigonometric functions*** plays an important role in such situations. Equations 1.4–1.6 give the inverse sine, inverse cosine, and inverse tangent in terms of the symbols used in the drawing. For instance, Equation 1.4 is read as "θ equals the angle whose sine is h_o/h."

$$\theta = \sin^{-1}\left(\frac{h_o}{h}\right) \tag{1.4}$$

$$\theta = \cos^{-1}\left(\frac{h_a}{h}\right) \tag{1.5}$$

$$\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right) \tag{1.6}$$

The use of -1 as an exponent in Equations 1.4–1.6 *does not mean* "take the reciprocal." For instance, $\tan^{-1}(h_o/h_a)$ does not equal $1/\tan(h_o/h_a)$. Another way to express the inverse trigonometric functions is to use arc sin, arc cos, and arc tan instead of $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$. Example 5 illustrates the use of an inverse trigonometric function.

Example 5 Using Inverse Trigonometric Functions

A lakefront drops off gradually at an angle θ, as Figure 1.7 indicates. For safety reasons, it is necessary to know how deep the lake is at various distances from the shore. To provide some information about the depth, a lifeguard rows straight out from the shore a distance of 14.0 m and drops a weighted fishing line. By measuring the length of the line, the lifeguard determines the depth to be 2.25 m. **(a)** What is the value of θ? **(b)** What would be the depth d of the lake at a distance of 22.0 m from the shore?

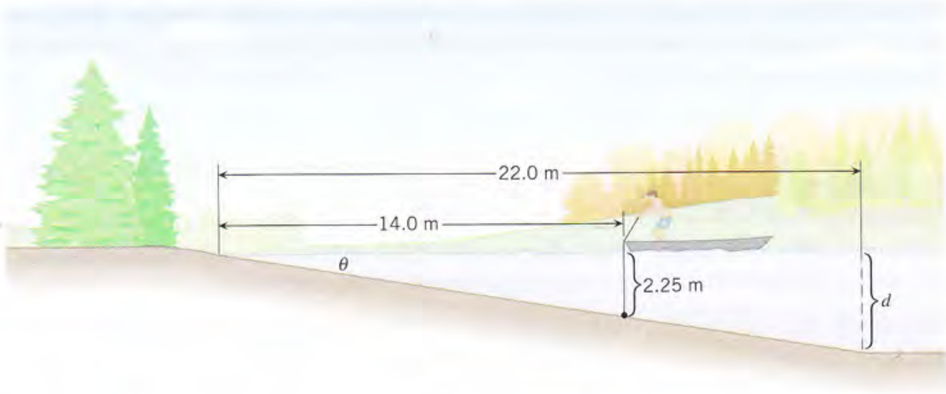


Figure 1.7 If the distance from the shore and the depth of the water at any one point are known, the angle θ can be found with the aid of trigonometry. Knowing the value of θ is useful, because then the depth d at another point can be determined.

Reasoning Near the shore, the lengths of the opposite and adjacent sides of the right triangle in Figure 1.7 are $h_o = 2.25$ m and $h_a = 14.0$ m, relative to the angle θ. Having made this identification, we can use the inverse tangent to find the angle in part (a). For part (b) the opposite and adjacent sides farther from the shore become $h_o = d$ and $h_a = 22.0$ m. With the value for θ obtained in part (a), the tangent function can be used to find the unknown depth. Considering the way in which the lake bottom drops off in Figure 1.7, we expect the unknown depth to be greater than 2.25 m.

Solution **(a)** Using the inverse tangent given in Equation 1.6, we find that

$$\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right) = \tan^{-1}\left(\frac{2.25\text{ m}}{14.0\text{ m}}\right) = \boxed{9.13^\circ}$$

(b) With $\theta = 9.13^\circ$, the tangent function given in Equation 1.3 can be used to find the unknown depth farther from the shore, where $h_o = d$ and $h_a = 22.0$ m. Since $\tan\theta = h_o/h_a$, it follows that

$$h_o = h_a \tan\theta$$
$$d = (22.0\text{ m})(\tan 9.13^\circ) = \boxed{3.54\text{ m}}$$

which is greater than 2.25 m, as expected.

The right triangle in Figure 1.5 provides the basis for defining the various trigonometric functions according to Equations 1.1–1.3. These functions always involve an angle and two sides of the triangle. There is also a relationship among the lengths of the three sides of a right triangle. This relationship is known as the ***Pythagorean theorem*** and is used often in this text.

Pythagorean Theorem

The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the other two sides:

$$h^2 = h_o^2 + h_a^2 \tag{1.7}$$

1.5 Scalars and Vectors

The volume of water in a swimming pool might be 50 cubic meters, or the winning time of a race could be 11.3 seconds. In cases like these, only the size of the numbers matters. In other words, *how much* volume or time is there? The 50 specifies the amount of water in units of cubic meters, while the 11.3 specifies the amount of time in seconds. Volume and time are examples of scalar quantities. A ***scalar quantity*** is one that can be described with a single number (including any units) giving its size or magnitude. Some other common scalars are temperature (e.g., 20 °C) and mass (e.g., 85 kg).

While many quantities in physics are scalars, there are also many that are not, and for these quantities the magnitude tells only part of the story. Consider Figure 1.8, which depicts a car that has moved 2 km along a straight line from start to finish. When describing the motion, it is incomplete to say that "the car moved a distance of 2 km." This statement would indicate only that the car ends up somewhere on a circle whose center is at the starting point and whose radius is 2 km. A complete description must include the direction along with the distance, as in the statement "the car moved a distance of 2 km in a direction 30° north of east." A quantity that deals inherently with *both magnitude and direction* is called a ***vector quantity.*** Because direction is an important characteristic of vectors, arrows are used to represent them; ***the direction of the arrow gives the direction of the vector.*** The colored arrow in Figure 1.8, for example, is called the *displacement vector,* because it shows how the car is displaced from its starting point. Chapter 2 discusses this particular vector.

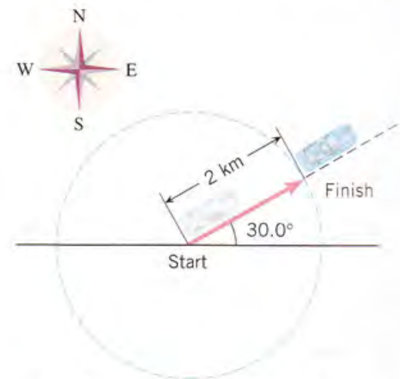


Figure 1.8 A vector quantity has a magnitude and a direction. The colored arrow in this drawing represents a displacement vector.

The length of the arrow in Figure 1.8 represents the magnitude of the displacement vector. If the car had moved 4 km instead of 2 km from the starting point, the arrow would have been drawn twice as long. ***By convention, the length of a vector arrow is proportional to the magnitude of the vector.***

In physics there are many important kinds of vectors, and the practice of using the length of an arrow to represent the magnitude of a vector applies to each of them. All forces, for instance, are vectors. In common usage a force is a push or a pull, and the direction in which a force acts is just as important as the strength or magnitude of the force. The magnitude of a force is measured in SI units called newtons (N). An arrow representing a force of 20 newtons is drawn twice as long as one representing a force of 10 newtons.

The fundamental distinction between scalars and vectors is the characteristic of direction. Vectors have it, and scalars do not. Conceptual Example 6 helps to clarify this distinction and explains what is meant by the "direction" of a vector.

Conceptual Example 6 Vectors, Scalars, and the Role of Plus and Minus Signs

There are places where the temperature is +20 °C at one time of the year and −20 °C at another time. Do the plus and minus signs that signify positive and negative temperatures imply that temperature is a vector quantity? **(a)** Yes **(b)** No

Reasoning A hallmark of a vector is that there is both a magnitude and a physical direction associated with it, such as 20 meters due east or 20 meters due west.

Answer (a) is incorrect. The plus and minus signs associated with +20 °C and −20 °C do not convey a physical direction, such as due east or due west. Therefore, temperature cannot be a vector quantity.

Answer (b) is correct. On a thermometer, the algebraic signs simply mean that the temperature is a number less than or greater than zero on the temperature scale being used and have nothing to do with east, west, or any other physical direction. Temperature, then, is not a vector. It is a scalar, and scalars can sometimes be negative.

The velocity of this cyclist is an example of a vector quantity, because it has a magnitude (his speed) and a direction. The cyclist is seven-time Tour-de-France winner Lance Armstrong. (© Steven E. Sutton/Duomo/Corbis)

Often, for the sake of convenience, quantities such as volume, time, displacement, velocity, and force are represented in physics by symbols. In this text, we write vectors in boldface symbols (**this is boldface**) with arrows above them* and write scalars in italic symbols (*this is italic*). Thus, a displacement vector is written as "$\vec{\mathbf{A}}$ = 750 m, due east," where the $\vec{\mathbf{A}}$ is a boldface symbol. By itself, however, separated from the direction, the magnitude of this vector is a scalar quantity. Therefore, the magnitude is written as "A = 750 m," where the A is an italic symbol without an arrow.

*Vectors are also sometimes written in other texts as boldface symbols without arrows above them.

Check Your Understanding

(The answer is given at the end of the book.)

5. Which of the following statements, if any, involves a vector? **(a)** I walked 2 miles along the beach. **(b)** I walked 2 miles due north along the beach. **(c)** I jumped off a cliff and hit the water traveling at 17 miles per hour. **(d)** I jumped off a cliff and hit the water traveling straight down at a speed of 17 miles per hour. **(e)** My bank account shows a negative balance of −25 dollars.

1.6 Vector Addition and Subtraction

■ Addition

Often it is necessary to add one vector to another, and the process of addition must take into account both the magnitude and the direction of the vectors. The simplest situation occurs when the vectors point along the same direction—that is, when they are colinear, as in Figure 1.9. Here, a car first moves along a straight line, with a displacement vector $\vec{\mathbf{A}}$ of 275 m, due east. Then the car moves again in the same direction, with a displacement vector $\vec{\mathbf{B}}$ of 125 m, due east. These two vectors add to give the total displacement vector $\vec{\mathbf{R}}$, which would apply if the car had moved from start to finish in one step. The symbol $\vec{\mathbf{R}}$ is used because the total vector is often called the ***resultant vector.*** With the tail of the second arrow located at the head of the first arrow, the two lengths simply add to give the length of the total displacement. This kind of vector addition is identical to the familiar addition of two scalar numbers ($2 + 3 = 5$) *and can be carried out here only because the vectors point along the same direction.* In such cases we add the individual magnitudes to get the magnitude of the total, knowing in advance what the direction must be. Formally, the addition is written as follows:

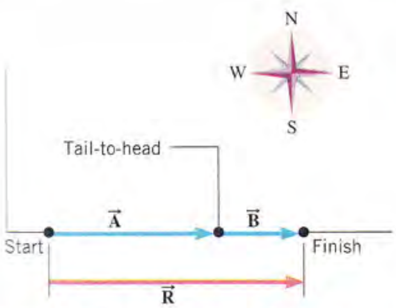


Figure 1.9 Two colinear displacement vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ add to give the resultant displacement vector $\vec{\mathbf{R}}$.

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$

$$\vec{\mathbf{R}} = 275 \text{ m, due east} + 125 \text{ m, due east} = 400 \text{ m, due east}$$

Perpendicular vectors are frequently encountered, and Figure 1.10 indicates how they can be added. This figure applies to a car that first travels with a displacement vector $\vec{\mathbf{A}}$ of 275 m, due east, and then with a displacement vector $\vec{\mathbf{B}}$ of 125 m, due north. The two vectors add to give a resultant displacement vector $\vec{\mathbf{R}}$. Once again, the vectors to be added are arranged in a tail-to-head fashion, and the resultant vector points from the tail of the first to the head of the last vector added. The resultant displacement is given by the vector equation

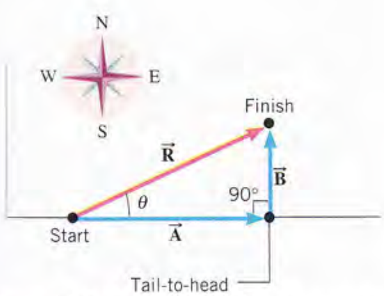


Figure 1.10 The addition of two perpendicular displacement vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ gives the resultant vector $\vec{\mathbf{R}}$.

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$

The addition in this equation cannot be carried out by writing $R = 275 \text{ m} + 125 \text{ m}$, because the vectors have different directions. Instead, we take advantage of the fact that the triangle in Figure 1.10 is a right triangle and use the Pythagorean theorem (Equation 1.7). According to this theorem, the magnitude of $\vec{\mathbf{R}}$ is

$$R = \sqrt{(275 \text{ m})^2 + (125 \text{ m})^2} = 302 \text{ m}$$

The angle θ in Figure 1.10 gives the direction of the resultant vector. Since the lengths of all three sides of the right triangle are now known, $\sin\theta$, $\cos\theta$, or $\tan\theta$ can be used to determine θ. Noting that $\tan\theta = B/A$ and using the inverse trigonometric function, we find that:

$$\theta = \tan^{-1}\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{125 \text{ m}}{275 \text{ m}}\right) = 24.4^\circ$$

Thus, the resultant displacement of the car has a magnitude of 302 m and points north of east at an angle of 24.4°. This displacement would bring the car from the start to the finish in Figure 1.10 in a single straight-line step.

When two vectors to be added are not perpendicular, the tail-to-head arrangement does not lead to a right triangle, and the Pythagorean theorem cannot be used. Figure 1.11*a* illustrates such a case for a car that moves with a displacement $\vec{\mathbf{A}}$ of 275 m, due east, and then with a displacement $\vec{\mathbf{B}}$ of 125 m, in a direction 55.0° north of west. As usual, the resultant displacement vector $\vec{\mathbf{R}}$ is directed from the tail of the first to the head of the last vector added. The vector addition is still given according to

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$

However, the magnitude of $\vec{\mathbf{R}}$ is not $R = A + B$, because the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ do not have the same direction, and neither is it $R = \sqrt{A^2 + B^2}$, because the vectors are not perpendicular, so the Pythagorean theorem does not apply. Some other means must be used to find the magnitude and direction of the resultant vector.

One approach uses a graphical technique. In this method, a diagram is constructed in which the arrows are drawn tail to head. The lengths of the vector arrows are drawn to scale, and the angles are drawn accurately (with a protractor, perhaps). Then the length of the arrow representing the resultant vector is measured with a ruler. This length is converted to the magnitude of the resultant vector by using the scale factor with which the drawing is constructed. In Figure 1.11*b*, for example, a scale of one centimeter of arrow length for each 10.0 m of displacement is used, and it can be seen that the length of the arrow representing $\vec{\mathbf{R}}$ is 22.8 cm. Since each centimeter corresponds to 10.0 m of displacement, the magnitude of $\vec{\mathbf{R}}$ is 228 m. The angle θ, which gives the direction of $\vec{\mathbf{R}}$, can be measured with a protractor to be $\theta = 26.7°$ north of east.

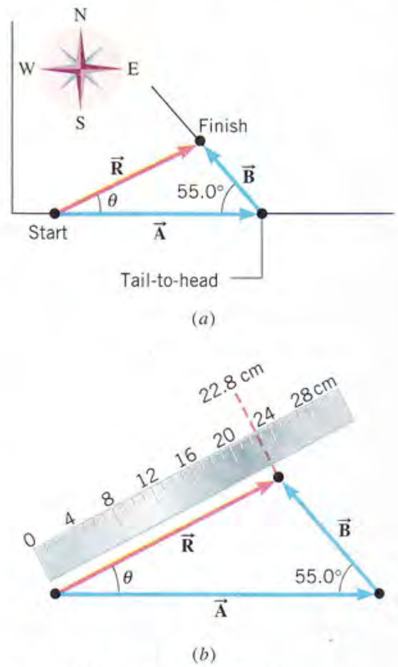


Figure 1.11 (*a*) The two displacement vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are neither colinear nor perpendicular, but even so they add to give the resultant vector $\vec{\mathbf{R}}$. (*b*) In one method for adding them together, a graphical technique is used.

Subtraction

The subtraction of one vector from another is carried out in a way that depends on the following fact. *When a vector is multiplied by −1, the magnitude of the vector remains the same, but the direction of the vector is reversed.* Conceptual Example 7 illustrates the meaning of this statement.

Conceptual Example 7 Multiplying a Vector by −1

Consider two vectors described as follows:

1. A woman climbs 1.2 m up a ladder, so that her displacement vector $\vec{\mathbf{D}}$ is 1.2 m, upward along the ladder, as in Figure 1.12*a*.
2. A man is pushing with 450 N of force on his stalled car, trying to move it eastward. The force vector $\vec{\mathbf{F}}$ that he applies to the car is 450 N, due east, as in Figure 1.13*a*.

What are the physical meanings of the vectors $-\vec{\mathbf{D}}$ and $-\vec{\mathbf{F}}$?

(a) $-\vec{\mathbf{D}}$ points upward along the ladder and has a magnitude of −1.2 m; $-\vec{\mathbf{F}}$ points due east and has a magnitude of −450 N. **(b)** $-\vec{\mathbf{D}}$ points downward along the ladder and has a magnitude of −1.2 m; $-\vec{\mathbf{F}}$ points due west and has a magnitude of −450 N. **(c)** $-\vec{\mathbf{D}}$ points downward along the ladder and has a magnitude of 1.2 m; $-\vec{\mathbf{F}}$ points due west and has a magnitude of 450 N.

Reasoning A displacement vector of $-\vec{\mathbf{D}}$ is $(-1)\,\vec{\mathbf{D}}$. The presence of the (−1) factor reverses the direction of the vector, but does not change its magnitude. Similarly, a force vector of $-\vec{\mathbf{F}}$ has the same magnitude as the vector $\vec{\mathbf{F}}$ but has the opposite direction.

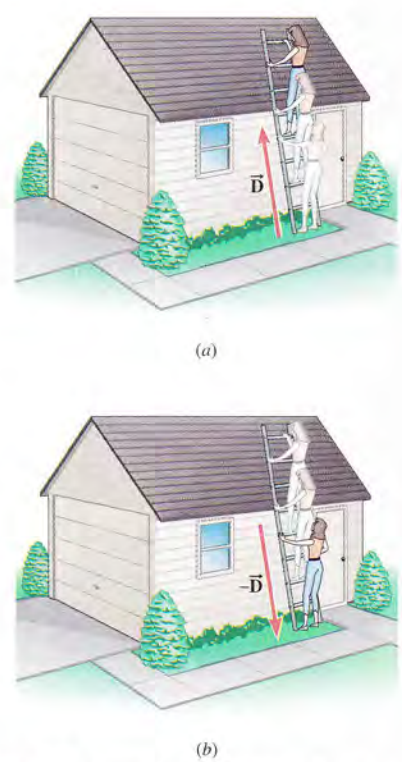


Figure 1.12 (*a*) The displacement vector for a woman climbing 1.2 m up a ladder is $\vec{\mathbf{D}}$. (*b*) The displacement vector for a woman climbing 1.2 m down a ladder is $-\vec{\mathbf{D}}$.

Figure 1.13 (*a*) The force vector for a man pushing on a car with 450 N of force in a direction due east is $\vec{\mathbf{F}}$. (*b*) The force vector for a man pushing on a car with 450 N of force in a direction due west is $-\vec{\mathbf{F}}$.

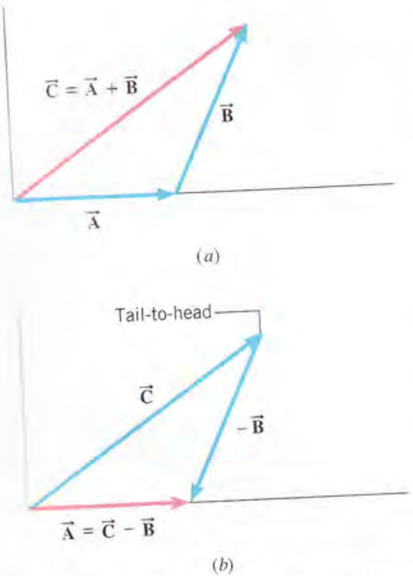


Figure 1.14 (*a*) Vector addition according to $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. (*b*) Vector subtraction according to $\vec{\mathbf{A}} = \vec{\mathbf{C}} - \vec{\mathbf{B}} = \vec{\mathbf{C}} + (-\vec{\mathbf{B}})$.

Answer (a) and (b) are incorrect. While scalars can sometimes be negative, magnitudes of vectors are never negative.

Answer (c) is correct. The vectors $-\vec{\mathbf{D}}$ and $-\vec{\mathbf{F}}$ have the same magnitudes as $\vec{\mathbf{D}}$ and $\vec{\mathbf{F}}$, but point in the opposite direction, as indicated in Figures 1.12*b* and 1.13*b*.

Related Homework: Problem 67

In practice, vector subtraction is carried out exactly like vector addition, except that one of the vectors added is multiplied by a scalar factor of -1. To see why, look at the two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ in Figure 1.14*a*. These vectors add together to give a third vector $\vec{\mathbf{C}}$, according to $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. Therefore, we can calculate vector $\vec{\mathbf{A}}$ as $\vec{\mathbf{A}} = \vec{\mathbf{C}} - \vec{\mathbf{B}}$, which is an example of vector subtraction. However, we can also write this result as $\vec{\mathbf{A}} = \vec{\mathbf{C}} + (-\vec{\mathbf{B}})$ and treat it as vector addition. Figure 1.14*b* shows how to calculate vector $\vec{\mathbf{A}}$ by adding the vectors $\vec{\mathbf{C}}$ and $-\vec{\mathbf{B}}$. Notice that vectors $\vec{\mathbf{C}}$ and $-\vec{\mathbf{B}}$ are arranged tail to head and that any suitable method of vector addition can be employed to determine $\vec{\mathbf{A}}$.

Check Your Understanding

(The answers are given at the end of the book.)

6. Two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are added together to give a resultant vector $\vec{\mathbf{R}}$: $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. The magnitudes of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are 3 m and 8 m, respectively, but the vectors can have any orientation. What are **(a)** the maximum possible value and **(b)** the minimum possible value for the magnitude of $\vec{\mathbf{R}}$?
7. Can two nonzero perpendicular vectors be added together so their sum is zero?
8. Can three or more vectors with unequal magnitudes be added together so their sum is zero?
9. In preparation for this question, review Conceptual Example 7. Vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ satisfy the vector equation $\vec{\mathbf{A}} + \vec{\mathbf{B}} = 0$. **(a)** How does the magnitude of $\vec{\mathbf{B}}$ compare with the magnitude of $\vec{\mathbf{A}}$? **(b)** How does the direction of $\vec{\mathbf{B}}$ compare with the direction of $\vec{\mathbf{A}}$?
10. Vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ satisfy the vector equation $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{C}}$, and their magnitudes are related by the scalar equation $A^2 + B^2 = C^2$. How is vector $\vec{\mathbf{A}}$ oriented with respect to vector $\vec{\mathbf{B}}$?
11. Vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ satisfy the vector equation $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{C}}$, and their magnitudes are related by the scalar equation $A + B = C$. How is vector $\vec{\mathbf{A}}$ oriented with respect to vector $\vec{\mathbf{B}}$?

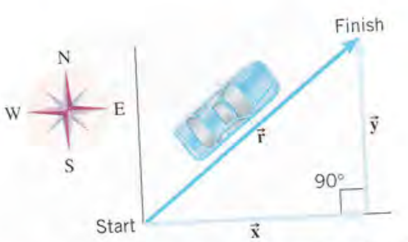


Figure 1.15 The displacement vector $\vec{\mathbf{r}}$ and its vector components $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$.

1.7 The Components of a Vector

Vector Components

Suppose a car moves along a straight line from start to finish, as in Figure 1.15, the corresponding displacement vector being $\vec{\mathbf{r}}$. The magnitude and direction of the vector $\vec{\mathbf{r}}$ give the distance and direction traveled along the straight line. However, the car could also arrive at the finish point by first moving due east, turning through 90°, and then moving due north. This alternative path is shown in the drawing and is associated with the two displacement vectors $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$. The vectors $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$ are called the *x vector component* and the *y vector component* of $\vec{\mathbf{r}}$.

Vector components are very important in physics and have two basic features that are apparent in Figure 1.15. One is that the components add together to equal the original vector:

$$\vec{\mathbf{r}} = \vec{\mathbf{x}} + \vec{\mathbf{y}}$$

The components $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$, when added vectorially, convey exactly the same meaning as does the original vector $\vec{\mathbf{r}}$: they indicate how the finish point is displaced relative to the starting point. The other feature of vector components that is apparent in Figure 1.15 is that $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$ are not just any two vectors that add together to give the original vector $\vec{\mathbf{r}}$: they are perpendicular vectors. This perpendicularity is a valuable characteristic, as we will soon see.

Any type of vector may be expressed in terms of its components, in a way similar to that illustrated for the displacement vector in Figure 1.15. Figure 1.16 shows an arbitrary vector $\vec{\mathbf{A}}$ and its vector components $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$. The components are drawn parallel to

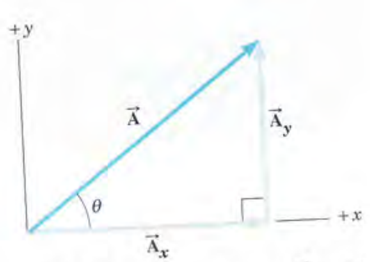


Figure 1.16 An arbitrary vector $\vec{\mathbf{A}}$ and its vector components $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$.

convenient x and y axes and are perpendicular. They add vectorially to equal the original vector $\vec{\mathbf{A}}$:

$$\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$$

There are times when a drawing such as Figure 1.16 is not the most convenient way to represent vector components, and Figure 1.17 presents an alternative method. The disadvantage of this alternative is that the tail-to-head arrangement of $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$ is missing, an arrangement that is a nice reminder that $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$ add together to equal $\vec{\mathbf{A}}$.

The definition that follows summarizes the meaning of vector components:

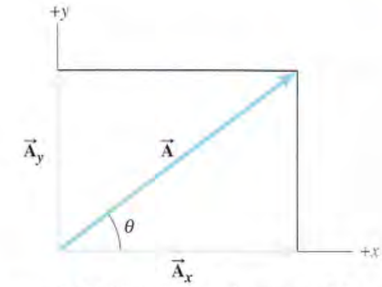


Figure 1.17 This alternative way of drawing the vector $\vec{\mathbf{A}}$ and its vector components is completely equivalent to that shown in Figure 1.16.

Definition of Vector Components

In two dimensions, the vector components of a vector $\vec{\mathbf{A}}$ are two perpendicular vectors $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$ that are parallel to the x and y axes, respectively, and add together vectorially according to $\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$.

Problem-Solving Insight. ***In general, the components of any vector can be used in place of the vector itself in any calculation where it is convenient to do so.*** The values calculated for vector components depend on the orientation of the vector relative to the axes used as a reference. Figure 1.18 illustrates this fact for a vector $\vec{\mathbf{A}}$ by showing two sets of axes, one set being rotated clockwise relative to the other. With respect to the black axes, vector $\vec{\mathbf{A}}$ has perpendicular vector components $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$; with respect to the colored rotated axes, vector $\vec{\mathbf{A}}$ has different vector components $\vec{\mathbf{A}}'_x$ and $\vec{\mathbf{A}}'_y$. The choice of which set of components to use is purely a matter of convenience.

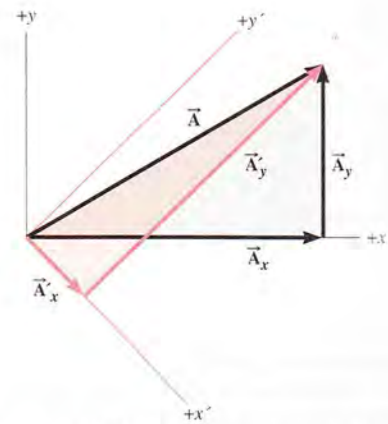


Figure 1.18 The vector components of the vector depend on the orientation of the axes used as a reference.

Scalar Components

It is often easier to work with the ***scalar components,*** A_x and A_y (note the italic symbols), rather than the vector components $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$. Scalar components are positive or negative numbers (with units) that are defined as follows: The scalar component A_x has a magnitude equal to that of $\vec{\mathbf{A}}_x$ and is given a positive sign if $\vec{\mathbf{A}}_x$ points along the $+x$ axis and a negative sign if $\vec{\mathbf{A}}_x$ points along the $-x$ axis. The scalar component A_y is defined in a similar manner. The following table shows an example of vector and scalar components:

Vector Components	Scalar Components	Unit Vectors
$\vec{\mathbf{A}}_x$ = 8 meters, directed along the $+x$ axis	$A_x = +8$ meters	$\vec{\mathbf{A}}_x = (+8 \text{ meters})\,\hat{\mathbf{x}}$
$\vec{\mathbf{A}}_y$ = 10 meters, directed along the $-y$ axis	$A_y = -10$ meters	$\vec{\mathbf{A}}_y = (-10 \text{ meters})\,\hat{\mathbf{y}}$

In this text, when we use the term "component," we will be referring to a scalar component, unless otherwise indicated.

Another method of expressing vector components is to use unit vectors. A ***unit vector*** is a vector that has a magnitude of 1, but no dimensions. We will use a caret (^) to distinguish it from other vectors. Thus,

$\hat{\mathbf{x}}$ is a dimensionless unit vector of length 1 that points in the positive x direction, and
$\hat{\mathbf{y}}$ is a dimensionless unit vector of length 1 that points in the positive y direction.

These unit vectors are illustrated in Figure 1.19. With the aid of unit vectors, the vector components of an arbitrary vector $\vec{\mathbf{A}}$ can be written as $\vec{\mathbf{A}}_x = A_x\,\hat{\mathbf{x}}$ and $\vec{\mathbf{A}}_y = A_y\,\hat{\mathbf{y}}$, where A_x and A_y are its scalar components (see the drawing and the third column of the table above). The vector $\vec{\mathbf{A}}$ is then written as $\vec{\mathbf{A}} = A_x\,\hat{\mathbf{x}} + A_y\,\hat{\mathbf{y}}$.

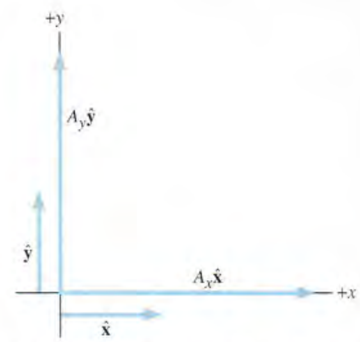


Figure 1.19 The dimensionless unit vectors $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ have magnitudes equal to 1, and they point in the $+x$ and $+y$ directions, respectively. Expressed in terms of unit vectors, the vector components of the vector $\vec{\mathbf{A}}$ are $A_x\,\hat{\mathbf{x}}$ and $A_y\,\hat{\mathbf{y}}$.

Resolving a Vector into Its Components

If the magnitude and direction of a vector are known, it is possible to find the components of the vector. The process of finding the components is called "resolving the vector into its components." As Example 8 illustrates, this process can be carried out with the aid of trigonometry, because the two perpendicular vector components and the original vector form a right triangle.

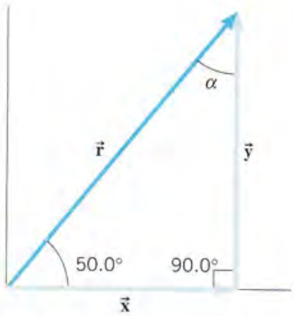


Figure 1.20 The x and y components of the displacement vector $\vec{\mathbf{r}}$ can be found using trigonometry.

Example 8 Finding the Components of a Vector

A displacement vector $\vec{\mathbf{r}}$ has a magnitude of $r = 175$ m and points at an angle of 50.0° relative to the x axis in Figure 1.20. Find the x and y components of this vector.

Reasoning We will base our solution on the fact that the triangle formed in Figure 1.20 by the vector $\vec{\mathbf{r}}$ and its components $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$ is a right triangle. This fact enables us to use the trigonometric sine and cosine functions, as defined in Equations 1.1 and 1.2.

Solution The y component can be obtained using the 50.0° angle and Equation 1.1, $\sin\theta = y/r$:

$$y = r\sin\theta = (175\text{ m})(\sin 50.0°) = \boxed{134\text{ m}}$$

In a similar fashion, the x component can be obtained using the 50.0° angle and Equation 1.2, $\cos\theta = x/r$:

$$x = r\cos\theta = (175\text{ m})(\cos 50.0°) = \boxed{112\text{ m}}$$

MATH SKILLS Either acute angle of a right triangle can be used to determine the components of a vector. The choice of angle is a matter of convenience. For instance, instead of the 50.0° angle, it is also possible to use the angle α in Figure 1.20. Since $\alpha + 50.0° = 90.0°$, it follows that $\alpha = 40.0°$. The solution using α yields the same answers as the solution using the 50.0° angle:

$$\cos\alpha = \frac{y}{r}$$

$$y = r\cos\alpha = (175\text{ m})(\cos 40.0°) = \boxed{134\text{ m}}$$

$$\sin\alpha = \frac{x}{r}$$

$$x = r\sin\alpha = (175\text{ m})(\sin 40.0°) = \boxed{112\text{ m}}$$

Problem-Solving Insight. You can check to see whether the components of a vector are correct by substituting them into the Pythagorean theorem in order to calculate the magnitude of the original vector.

Since the vector components and the original vector form a right triangle, the Pythagorean theorem can be applied to check the validity of calculations such as those in Example 8. Thus, with the components obtained in Example 8, the theorem can be used to verify that the magnitude of the original vector is indeed 175 m, as given initially:

$$r = \sqrt{(112\text{ m})^2 + (134\text{ m})^2} = 175\text{ m}$$

Problem-Solving Insight.

It is possible for one of the components of a vector to be zero. This does not mean that the vector itself is zero, however. ***For a vector to be zero, every vector component must individually be zero.*** Thus, in two dimensions, saying that $\vec{\mathbf{A}} = \mathbf{0}$ is equivalent to saying that $\vec{\mathbf{A}}_x = \mathbf{0}$ and $\vec{\mathbf{A}}_y = \mathbf{0}$. Or, stated in terms of scalar components, if $\vec{\mathbf{A}} = \mathbf{0}$, then $A_x = 0$ and $A_y = 0$.

Problem-Solving Insight.

Two vectors are equal if, and only if, they have the same magnitude and direction. Thus, if one displacement vector points east and another points north, they are *not* equal, even if each has the same magnitude of 480 m. In terms of vector components, two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are equal if, and only if, each vector component of one is equal to the corresponding vector component of the other. In two dimensions, if $\vec{\mathbf{A}} = \vec{\mathbf{B}}$, then $\vec{\mathbf{A}}_x = \vec{\mathbf{B}}_x$ and $\vec{\mathbf{A}}_y = \vec{\mathbf{B}}_y$. Alternatively, using scalar components, we write that $A_x = B_x$ and $A_y = B_y$.

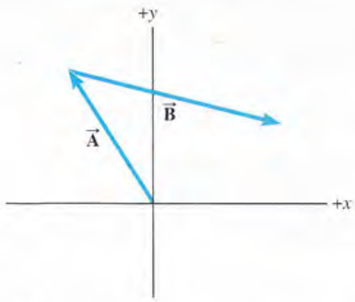
Check Your Understanding

(The answers are given at the end of the book.)

12. Which of the following displacement vectors (if any) are equal?

Variable	Magnitude	Direction
$\vec{\mathbf{A}}$	100 m	30° north of east
$\vec{\mathbf{B}}$	100 m	30° south of west
$\vec{\mathbf{C}}$	50 m	30° south of west
$\vec{\mathbf{D}}$	100 m	60° east of north

13. Two vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, are shown in the drawing. **(a)** What are the signs (+ or −) of the scalar components, A_x and A_y, of the vector $\vec{\mathbf{A}}$? **(b)** What are the signs of the scalar components, B_x and B_y, of the vector $\vec{\mathbf{B}}$? **(c)** What are the signs of the scalar components, R_x and R_y, of the vector $\vec{\mathbf{R}}$, where $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$?



14. Are two vectors with the same magnitude necessarily equal?
15. The magnitude of a vector has doubled, its direction remaining the same. Can you conclude that the magnitude of each component of the vector has doubled?
16. The tail of a vector is fixed to the origin of an x, y axis system. Originally the vector points along the $+x$ axis and has a magnitude of 12 units. As time passes, the vector rotates counterclockwise. What are the sizes of the x and y components of the vector for the following rotational angles? **(a)** 90° **(b)** 180° **(c)** 270° **(d)** 360°
17. A vector has a component of zero along the x axis of a certain axis system. Does this vector necessarily have a component of zero along the x axis of another (rotated) axis system?

1.8 Addition of Vectors by Means of Components

The components of a vector provide the most convenient and accurate way of adding (or subtracting) any number of vectors. For example, suppose that vector $\vec{\mathbf{A}}$ is added to vector $\vec{\mathbf{B}}$. The resultant vector is $\vec{\mathbf{C}}$, where $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. Figure 1.21*a* illustrates this vector addition, along with the x and y vector components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. In part *b* of the drawing, the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have been removed, because we can use the vector components of these vectors in place of them. The vector component $\vec{\mathbf{B}}_x$ has been shifted downward and arranged tail to head with vector component $\vec{\mathbf{A}}_x$. Similarly, the vector component $\vec{\mathbf{A}}_y$ has been shifted to the right and arranged tail to head with the vector component $\vec{\mathbf{B}}_y$. The x components are colinear and add together to give the x component of the resultant vector $\vec{\mathbf{C}}$. In like fashion, the y components are colinear and add together to give the y component of $\vec{\mathbf{C}}$. In terms of scalar components, we can write

$$C_x = A_x + B_x \quad \text{and} \quad C_y = A_y + B_y$$

The vector components $\vec{\mathbf{C}}_x$ and $\vec{\mathbf{C}}_y$ of the resultant vector form the sides of the right triangle shown in Figure 1.21*c*. Thus, we can find the magnitude of $\vec{\mathbf{C}}$ by using the Pythagorean theorem:

$$C = \sqrt{C_x^2 + C_y^2}$$

The angle θ that $\vec{\mathbf{C}}$ makes with the x axis is given by $\theta = \tan^{-1}(C_y/C_x)$. Example 9 illustrates how to add several vectors using the component method.

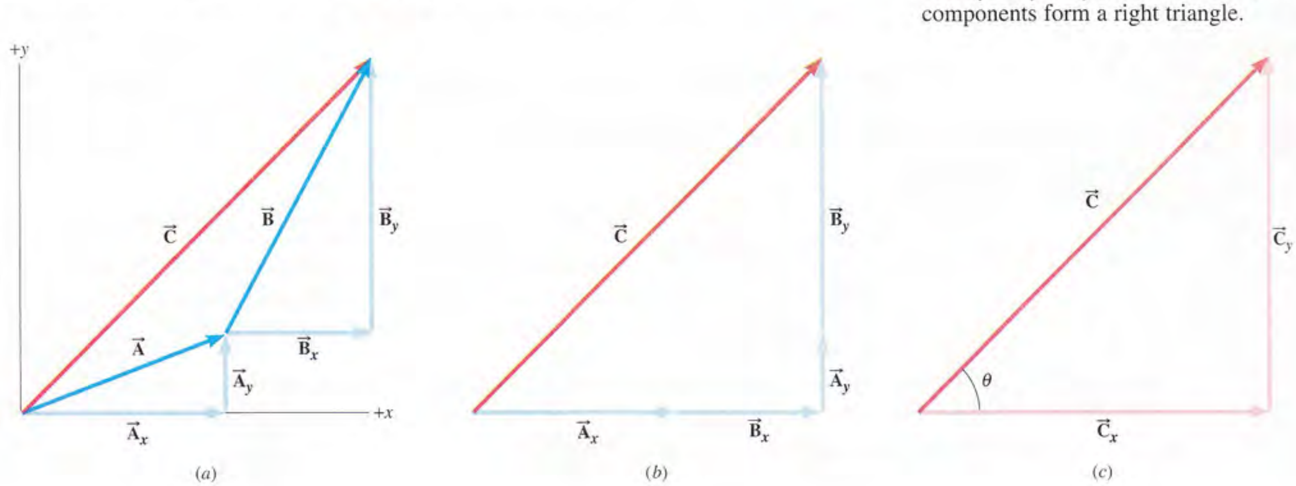


Figure 1.21 (*a*) The vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ add together to give the resultant vector $\vec{\mathbf{C}}$. The x and y components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are also shown. (*b*) The drawing illustrates that $\vec{\mathbf{C}}_x = \vec{\mathbf{A}}_x + \vec{\mathbf{B}}_x$ and $\vec{\mathbf{C}}_y = \vec{\mathbf{A}}_y + \vec{\mathbf{B}}_y$. (*c*) Vector $\vec{\mathbf{C}}$ and its components form a right triangle.

Analyzing Multiple-Concept Problems

Example 9 The Component Method of Vector Addition

A jogger runs 145 m in a direction 20.0° east of north (displacement vector $\vec{A}$) and then 105 m in a direction 35.0° south of east (displacement vector $\vec{B}$). Using components, determine the magnitude and direction of the resultant vector $\vec{C}$ for these two displacements.

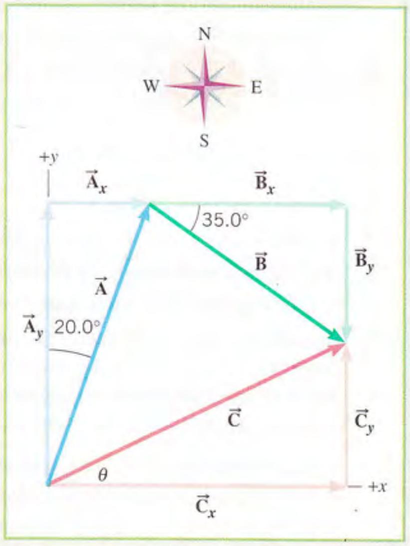


Figure 1.22 The vectors $\vec{A}$ and $\vec{B}$ add together to give the resultant vector $\vec{C}$. The vector components of $\vec{A}$ and $\vec{B}$ are also shown. The resultant vector $\vec{C}$ can be obtained once its components have been found.

Reasoning Figure 1.22 shows the vectors $\vec{A}$ and $\vec{B}$, assuming that the y axis corresponds to the direction due north. The vectors are arranged in a tail-to-head fashion, with the resultant vector $\vec{C}$ drawn from the tail of $\vec{A}$ to the head of $\vec{B}$. The components of the vectors are also shown in the figure. Since $\vec{C}$ and its components form a right triangle (red in the drawing), we will use the Pythagorean theorem and trigonometry to express the magnitude and directional angle θ for $\vec{C}$ in terms of its components. The components of $\vec{C}$ will then be obtained from the components of $\vec{A}$ and $\vec{B}$ and the data given for these two vectors.

Knowns and Unknowns The data for this problem are listed in the table that follows:

Description	Symbol	Value	Comment
Magnitude of vector $\vec{A}$		145 m	
Direction of vector $\vec{A}$		20.0° east of north	See Figure 1.22.
Magnitude of vector $\vec{B}$		105 m	
Direction of vector $\vec{B}$		35.0° south of east	See Figure 1.22.
Unknown Variables			
Magnitude of resultant vector	C	?	
Direction of resultant vector	θ	?	

Modeling the Problem

STEP 1 Magnitude and Direction of $\vec{C}$ In Figure 1.22 the vector $\vec{C}$ and its components $\vec{C}_x$ and $\vec{C}_y$ form a right triangle, as the red arrows show. Applying the Pythagorean theorem to this right triangle shows that the magnitude of $\vec{C}$ is given by Equation 1a at the right. From the red triangle it also follows that the directional angle θ for the vector $\vec{C}$ is given by Equation 1b at the right.

$$C = \sqrt{C_x^2 + C_y^2} \qquad (1a)$$

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) \qquad (1b)$$

STEP 2 Components of $\vec{C}$ Since vector $\vec{C}$ is the resultant of vectors $\vec{A}$ and $\vec{B}$, we have $\vec{C} = \vec{A} + \vec{B}$ and can write the scalar components of $\vec{C}$ as the sum of the scalar components of $\vec{A}$ and $\vec{B}$:

$$\boxed{C_x = A_x + B_x} \quad \text{and} \quad \boxed{C_y = A_y + B_y}$$

These expressions can be substituted into Equations 1a and 1b for the magnitude and direction of $\vec{C}$, as shown at the right.

$$C = \sqrt{C_x^2 + C_y^2} \qquad (1a)$$

$$\boxed{C_x = A_x + B_x} \quad \boxed{C_y = A_y + B_y} \qquad (2)$$

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) \qquad (1b)$$

$$\boxed{C_x = A_x + B_x} \quad \boxed{C_y = A_y + B_y} \qquad (2)$$

Solution Algebraically combining the results of each step, we find that

STEP 1 STEP 2

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{A_y + B_y}{A_x + B_x}\right)$$

STEP 1 STEP 2

To use these results we need values for the individual components of $\vec{A}$ and $\vec{B}$.

Referring to Figure 1.22, we find these values to be

$A_x = (145\text{ m})\sin 20.0° = 49.6\text{ m}$ and
$A_y = (145\text{ m})\cos 20.0° = 136\text{ m}$

$B_x = (105\text{ m})\cos 35.0° = 86.0\text{ m}$ and
$B_y = -(105\text{ m})\sin 35.0° = -60.2\text{ m}$

Note that the component B_y is negative, because $\vec{\mathbf{B}}_y$ points downward, in the negative y direction in the drawing. Substituting these values into the results for C and θ gives

$$C = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$
$$= \sqrt{(49.6\text{ m} + 86.0\text{ m})^2 + (136\text{ m} - 60.2\text{ m})^2} = \boxed{155\text{ m}}$$
$$\theta = \tan^{-1}\left(\frac{A_y + B_y}{A_x + B_x}\right)$$
$$= \tan^{-1}\left(\frac{136\text{ m} - 60.2\text{ m}}{49.6\text{ m} + 86.0\text{ m}}\right) = \boxed{29°}$$

Related Homework: ***Problems 45, 47, 50, 54***

MATH SKILLS According to the definitions given in Equations 1.1 and 1.2, the sine and cosine functions are $\sin\phi = \frac{h_o}{h}$ and $\cos\phi = \frac{h_a}{h}$, where h_o is the length of the side of a right triangle that is opposite the angle ϕ, h_a is the length of the side adjacent to the angle ϕ, and h is the length of the hypotenuse (see Figure 1.23*a*). Applications of the sine and cosine functions to determine the scalar components of a vector occur frequently. In such applications we begin by identifying the angle ϕ. Figure 1.23*b* shows the relevant portion of Figure 1.22 and indicates that $\phi = 20.0°$ for the vector $\vec{\mathbf{A}}$. In this case we have $h_o = A_x$, $h_a = A_y$, and $h = A = 145$ m; it follows that

$$\sin 20.0° = \frac{h_o}{h} = \frac{A_x}{A} \quad \text{or} \quad A_x = A\sin 20.0° = (145\text{ m})\sin 20.0° = 49.6\text{ m}$$

$$\cos 20.0° = \frac{h_a}{h} = \frac{A_y}{A} \quad \text{or} \quad A_y = A\cos 20.0° = (145\text{ m})\cos 20.0° = 136\text{ m}$$

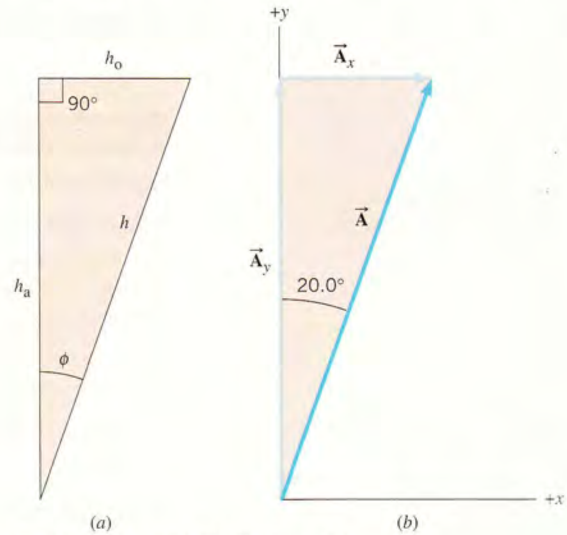
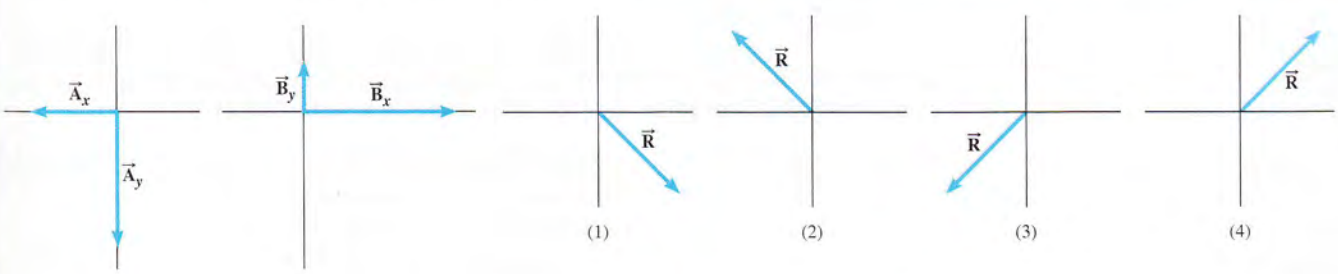


Figure 1.23 Math Skills drawing.

Check Your Understanding

(*The answer is given at the end of the book.*)

18. Two vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, have vector components that are shown (to the same scale) in the drawing. The resultant vector is labeled $\vec{\mathbf{R}}$. Which drawing shows the correct vector sum of $\vec{\mathbf{A}} + \vec{\mathbf{B}}$? **(a)** 1, **(b)** 2, **(c)** 3, **(d)** 4



In later chapters we will often use the component method for vector addition. For future reference, the main features of the reasoning strategy used in this technique are summarized below.

Reasoning Strategy The Component Method of Vector Addition

1. For each vector to be added, determine the x and y components relative to a conveniently chosen x, y coordinate system. Be sure to take into account the directions of the components by using plus and minus signs to denote whether the components point along the positive or negative axes.
2. Find the algebraic sum of the x components, which is the x component of the resultant vector. Similarly, find the algebraic sum of the y components, which is the y component of the resultant vector.

Continued

3. Use the x and y components of the resultant vector and the Pythagorean theorem to determine the magnitude of the resultant vector.
4. Use the inverse sine, inverse cosine, or inverse tangent function to find the angle that specifies the direction of the resultant vector.

1.9 Concepts & Calculations

This chapter has presented an introduction to the mathematics of trigonometry and vectors, which will be used throughout this text. Therefore, in this last section we consider several examples in order to review some of the important features of this mathematics. The three-part format of these examples stresses the role of conceptual understanding in problem solving. First, the problem statement is given. Then, there is a concept question-and-answer section, which is followed by the solution section. The purpose of the concept question-and-answer section is to provide help in understanding the solution and to illustrate how a review of the concepts can help in anticipating some of the characteristics of the numerical answers.

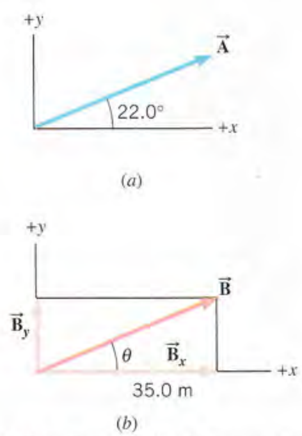


Figure 1.24 The two displacement vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are equal. Example 10 discusses what this equality means.

Concepts & Calculations Example 10

Equal Vectors

Figure 1.24 shows two displacement vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Vector $\vec{\mathbf{A}}$ points at an angle of 22.0° above the x axis and has an unknown magnitude. Vector $\vec{\mathbf{B}}$ has an x component $B_x = 35.0$ m and has an unknown y component B_y. These two vectors are equal. Find the magnitude of $\vec{\mathbf{A}}$ and the value of B_y.

Concept Questions and Answers What does the fact that vector $\vec{\mathbf{A}}$ equals vector $\vec{\mathbf{B}}$ imply about the magnitudes and directions of the vectors?

Answer When two vectors are equal, each has the same magnitude and each has the same direction.

What does the fact that vector $\vec{\mathbf{A}}$ equals vector $\vec{\mathbf{B}}$ imply about the x and y components of the vectors?

Answer When two vectors are equal, the x component of vector $\vec{\mathbf{A}}$ equals the x component of vector $\vec{\mathbf{B}}$ ($A_x = B_x$) and the y component of vector $\vec{\mathbf{A}}$ equals the y component of vector $\vec{\mathbf{B}}$ ($A_y = B_y$).

Solution We focus on the fact that the x components of the vectors are the same and the y components of the vectors are the same. This allows us to write that

$$\underbrace{A \cos 22.0^\circ}_{\substack{\text{Component } A_x \\ \text{of vector } \vec{\mathbf{A}}}} = \underbrace{35.0 \text{ m}}_{\substack{\text{Component } B_x \\ \text{of vector } \vec{\mathbf{B}}}} \tag{1.8}$$

$$\underbrace{A \sin 22.0^\circ}_{\substack{\text{Component } A_y \\ \text{of vector } \vec{\mathbf{A}}}} = \underbrace{B_y}_{\substack{\text{Component } B_y \\ \text{of vector } \vec{\mathbf{B}}}} \tag{1.9}$$

Dividing Equation 1.9 by Equation 1.8 shows that

$$\frac{A \sin 22.0^\circ}{A \cos 22.0^\circ} = \frac{B_y}{35.0 \text{ m}}$$

$$B_y = (35.0 \text{ m}) \frac{\sin 22.0^\circ}{\cos 22.0^\circ} = (35.0 \text{ m}) \tan 22.0^\circ = \boxed{14.1 \text{ m}}$$

Solving Equation 1.8 directly for A gives

$$A = \frac{35.0 \text{ m}}{\cos 22.0^\circ} = \boxed{37.7 \text{ m}}$$

Concepts & Calculations Example 11

Using Components to Add Vectors

Figure 1.25 shows three displacement vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$. These vectors are arranged in tail-to-head fashion, because they add together to give a resultant displacement $\vec{\mathbf{R}}$, which lies along the x axis. Note that the vector $\vec{\mathbf{B}}$ is parallel to the x axis. What is the magnitude of the vector $\vec{\mathbf{A}}$ and its directional angle θ?

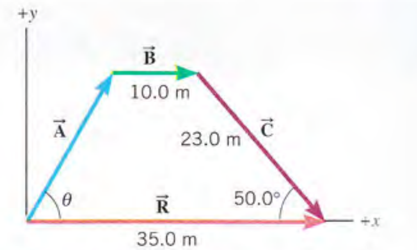


Figure 1.25 The displacement vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ add together to give the resultant displacement $\vec{\mathbf{R}}$, which lies along the x axis. Note that the vector $\vec{\mathbf{B}}$ is parallel to the x axis.

Concept Questions and Answers How is the magnitude of $\vec{\mathbf{A}}$ related to its scalar components A_x and A_y?

> *Answer* The magnitude of $\vec{\mathbf{A}}$ is given by the Pythagorean theorem in the form $A = \sqrt{A_x^2 + A_y^2}$, since a vector and its components form a right triangle (see Figure 1.26).

Do any of the vectors in Figure 1.25 have a zero value for either their x or y components?

> *Answer* Yes. The vectors $\vec{\mathbf{B}}$ and $\vec{\mathbf{R}}$ each have a zero value for their y component. This is because these vectors are parallel to the x axis.

What does the fact that $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ add together to give the resultant $\vec{\mathbf{R}}$ tell you about the components of these vectors?

> *Answer* The fact that $\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = \vec{\mathbf{R}}$ means that the sum of the x components of $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ equals the x component of $\vec{\mathbf{R}}$ ($A_x + B_x + C_x = R_x$) and the sum of the y components of $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ equals the y component of $\vec{\mathbf{R}}$ ($A_y + B_y + C_y = R_y$).

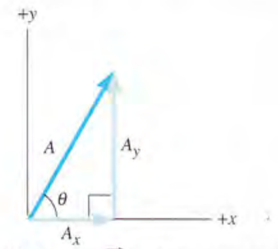


Figure 1.26 Vector $\vec{\mathbf{A}}$ and its components A_x and A_y are shown.

Solution To begin with, we apply the Pythagorean theorem to relate A, the magnitude of the vector $\vec{\mathbf{A}}$, to its scalar components A_x and A_y:

$$A = \sqrt{A_x^2 + A_y^2}$$

To obtain a value for A_x we use the fact that the sum of the x components of $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ equals the x component of $\vec{\mathbf{R}}$:

$$A_x + \underbrace{10.0\text{ m}}_{B_x} + \underbrace{(23.0\text{ m})\cos 50.0^\circ}_{C_x} = \underbrace{35.0\text{ m}}_{R_x} \quad \text{or} \quad A_x = 10.22\text{ m}$$

To obtain a value for A_y we note that the sum of the y components of $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ equals the y component of $\vec{\mathbf{R}}$ (and remember that $\vec{\mathbf{B}}$ and $\vec{\mathbf{R}}$ have no y components):

$$A_y + \underbrace{0\text{ m}}_{B_y} + \underbrace{[-(23.0\text{ m})\sin 50.0^\circ]}_{C_y} = \underbrace{0\text{ m}}_{R_y} \quad \text{or} \quad A_y = 17.62\text{ m}$$

With these values for A_x and A_y we find that the magnitude of $\vec{\mathbf{A}}$ is

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(10.22\text{ m})^2 + (17.62\text{ m})^2} = 20.37\text{ m} = \boxed{20.4\text{ m}}$$

The values for A_x and A_y can also be used to find θ. Referring to Figure 1.26, we see that $\tan\theta = A_y/A_x$. Therefore, we have

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{17.62\text{ m}}{10.22\text{ m}}\right) = \boxed{59.9^\circ}$$

MATH SKILLS Using the inverse tangent function is not the only way to determine the angle θ. For instance, it is also possible to use the inverse cosine function. Referring to Figure 1.26, we find that

$$\cos\theta = \frac{A_x}{A} \quad \text{or} \quad \theta = \cos^{-1}\left(\frac{A_x}{A}\right) = \cos^{-1}\left(\frac{10.22\text{ m}}{20.37\text{ m}}\right) = \boxed{59.9^\circ}$$

Another possibility is to use the inverse sine function. Referring again to Figure 1.26, we see that

$$\sin\theta = \frac{A_y}{A} \quad \text{or} \quad \theta = \sin^{-1}\left(\frac{A_y}{A}\right) = \sin^{-1}\left(\frac{17.62\text{ m}}{20.37\text{ m}}\right) = \boxed{59.9^\circ}$$

1 1.1.1. Which of the following individuals did not make significant contributions in physics?

a) Galileo Galilei

b) Isaac Newton

c) James Clerk Maxwell

d) Neville Chamberlain

2 1.1.2. Which of the following statements is not a reason that physics is a required course for students in a wide variety of disciplines?

a) There are usually not enough courses for students to take.

b) Students can learn to think like physicists.

c) Students can learn to apply physics principles to a wide range of problems.

d) Physics is both fascinating and fundamental.

e) Physics has important things to say about our environment.

3 1.2.1. When we measure physical quantities, the units may be anything that is reasonable as long as they are well defined. It’s usually best to use the international standard units. Density may be defined as the mass of an object divided by its volume. Which of the following units would probably not be acceptable units of density?

a) gallons/liter

b) kilograms/m^3

c) pounds/ft^3

d) slugs/yd^3

e) grams/milliliter

4 1.2.1. The text uses SI units. What do the “S” and the “I” stand for?

a) Système International

b) Science Institute

c) Swiss Institute

d) Systematic Information

e) Strong Interaction

5 1.2.2. Which of the following units is not an SI base unit?

a) slug

b) meter

c) kilogram

d) second

6 1.2.2. A car starts from rest on a circular track with a radius of 150 m. Relative to the starting position, what angle has the car swept out when it has traveled 150 m along the circular track?

a) 1 radian

b) $\pi/2$ radians

c) π radians

d) $3\pi/2$ radians

e) 2π radians

7 1.2.3. Complete the following statement: The standard meter is defined in terms of the speed of light because

a) all scientists have access to sunlight.

b) no agreement could be reached on a standard meter stick.

c) the yard is defined in terms of the speed of sound in air.

d) the normal meter is defined with respect to the circumference of the earth.

e) it is a universal constant.

8 1.2.3. The radius of sphere A is one half that of sphere B. How do the circumference and volume of sphere B compare to sphere A?

a) The circumference of B is 2 times that of A; and its volume is 4 times that of A.

b) The circumference of B is 2 times that of A; and its volume is 6 times that of A.

c) The circumference of B is 2 times that of A; and its volume is 8 times that of A.

d) The circumference of B is 4 times that of A; and its volume is 6 times that of A.

e) The circumference of B is 4 times that of A; and its volume is 8 times that of A.

9 1.2.4. Express 0.00592 in scientific notation.

a) 5.92×10^{3}

b) 5.92×10^{-3}

c) 5.92×10^{-2}

d) 5.92×10^{-5}

e) 5.92×10^{5}

10 1.2.4. A section of a river can be approximated as a rectangle that is 48 m wide and 172 m long. Express the area of this river in square kilometers.

a) 8.26×10^{-3} km^2

b) 8.26 km^2

c) 8.26×10^{3} km^2

d) 3.58 km^2

e) 3.58×10^{-2} km^2

11 1.2.5. The ratio $\frac{\text{one millimeter}}{\text{one kilometer}}$ equals

a) 10^{+3}

b) 10^{-3}

c) 10^{-6}

d) 10^{+6}

e) 10^{0}

12 1.2.5. Express the following statement as an algebraic expression: "There are 264 gallons in a one cubic meter container." Let G represent the number of gallons and M represent the number of one cubic meter containers.

a) $G = 264M$

b) $G = M/264$

c) $G = 0.00379M$

d) $M = G/264$

e) $M = G$

13 1.2.6. In the International System of Units, mass is measured using which of the following units?

a) grams

b) kilograms

c) pounds

d) newtons

e) slugs

14 1.2.6. If one inch is equal to 2.54 cm, express 9.68 inches in meters.

a) 0.262 m

b) 0.0381 m

c) 0.0508 m

d) 0.114 m

e) 0.246 m

15 1.2.7. In the International System of Units, length is measured using which of the following units?

a) inches

b) feet

c) meters

d) centimeters

e) kilometers

16 1.2.8. How many meters are there in 12.5 kilometers?

a) 1.25

b) 125

c) 1250

d) 12 500

e) 125 000

17 1.2.9 Express the quantity 12.5 meters in kilometers?

a) 0.0125 km

b) 0.125 km

c) 1.25 km

d) 12.5 km

e) 125 km

18 1.2.10. By international agreement, the standard meter is currently defined by which of the following methods.

a) The standard meter is one-ten millionth of the distance between the Equator and the North Pole.

b) The standard meter is the length of the path traveled by light in a vacuum during a specific time interval.

c) The standard meter is the distance between two fine parallel lines on a platinum bar stored under vacuum near Paris, France.

d) The standard meter is defined in terms of a specific number of wavelengths of light emitted by a specific isotope of an inert gas.

e) The standard meter is defined in terms of the length of the tibia bone of a 17th century king.

19 1.2.11. How is the standard unit of time, the “second,” defined in the International System of Units?

a) using the frequency of the light emitted from the ideal gas krypton

b) using a standard pendulum that has a length of exactly one standard meter

c) using a portion of the time for a single rotation of the Earth

d) using a high precision telescope to measure the light coming from the most distant objects in the Universe

e) using a high precision cesium (atomic) clock

20 1.3.1. Which one of the following statements concerning unit conversion is false?

a) Units can be treated as algebraic quantities.

b) Units have no numerical significance, so 1.00 kilogram = 1.00 slug.

c) Unit conversion factors are given inside the front cover of the text.

d) The fact that multiplying an equation by a factor of 1 does not change an equation is important in unit conversion.

e) Only quantities with the same units can be added or subtracted.

21 1.3.1. Using the dimensions given for the variables in the table, determine which one of the following expressions is correct.

variable	dimension
f	$\frac{1}{[T]}$
l	$[L]$
g	$\frac{[L]}{[T]^2}$

a) $f = \dfrac{g}{2\pi l}$

b) $f = 2\text{p}lg$

c) $2\pi f = \sqrt{\dfrac{g}{l}}$

d) $2\pi f = \sqrt{\dfrac{l}{g}}$

e) $f = 2\pi\sqrt{gl}$

22 1.3.2. Given the following equation: $y = c^n at^2$, where n is an integer with no units, c is a number between zero and one with no units, the variable t has units of seconds and y is expressed in meters, determine which of the following statements is true.

a) Through dimensional analysis, a has units of meters per second (m/s) and $n = 1$.

b) Through dimensional analysis, a has units of meters per second (m/s) and $n = 2$.

c) Through dimensional analysis, a has units of meters per second per second (m/s^2) and $n = 1$.

d) Through dimensional analysis, a has units of meters per second per second (m/s^2) and $n = 2$.

e) a has units of meters per second per second (m/s^2), but value of n cannot be determined through dimensional analysis.

23 1.3.2. Which one of the following pairs of units may not be added together, even after the appropriate unit conversions have been made?

a) feet and centimeters

b) seconds and slugs

c) meters and miles

d) grams and kilograms

e) hours and years

24 1.3.3. Which one of the following terms is used to refer to the physical nature of a quantity and the type of unit used to specify it?

a) scalar

b) conversion

c) dimension

d) vector

e) symmetry

25 1.3.3. Given the mathematical expression $A = 3B$, determine which of the following statements is true.

a) The ratio $B/A = 3$.

b) A is three times smaller than B.

c) The ratio A/B = 1/3.

d) B is three times smaller than A.

e) A and B can have any value. There is no relationship between them.

26 1.3.4. Complete the following statement: The ratio $\frac{1\,\text{milligram}}{1\,\text{kilogram}}$ is equal to

a) 10^{2}

b) 10^{3}

c) 10^{6}

d) 10^{-3}

e) 10^{-6}

27 1.3.4. In dimensional analysis, the dimensions for speed are

a) $\frac{[L]^2}{[T]}$

b) $\frac{[L]}{[T]^2}$

c) $\frac{[L]^2}{[T]^2}$

d) $\frac{[L]}{[T]}$

e) $\frac{[L]}{\sqrt{[T]}}$

28 1.3.5. Given that one inch is equal to 25.4 mm. How many feet equal 21 m?

a) 53.0 ft

b) 82.7 ft

c) 63.0 ft

d) 68.9 ft

e) 47.9 ft

29 1.3.6. Approximately how many seconds are there in a century?

a) 86 400 s

b) 5.0×10^6 s

c) 3.3×101^8 s

d) 3.2×10^9 s

e) 8.6×10^4 s

30 1.3.7. A glacier is receding at a constant rate of 0.4 cm per day. After 3.0 years, by what length has the glacier receded? Express your answer in meters.

a) 17.1 m

b) 4.38 m

c) 9.82 m

d) 1.46 m

e) 12.7 m

31 1.3.8. Consider each of the following comparisons between various length units. Which one of these comparisons is false?

a) 1 m > 10 cm

b) 1000 nm < 1 mm

c) 1 foot < 10 cm

d) 1 mile > 1 km

e) 10 inches < 1 m

32 1.3.9. Consider each of the following comparisons between various time units. Which one of these comparisons is false?

a) 84 600 s = 1 day

b) 1 h > 3000 s

c) 1 ns > 1000 μs

d) 1 s = 1000 ms

e) 1 y = 5.26×10^5 h

33 1.4.1. Which one of the following terms is not a trigonometric function?

a) cosine

b) tangent

c) sine

d) hypotenuse

e) arc tangent

34 1.4.1. The city of Denver is located approximately one mile (1.61 km) above sea level. Assume you are standing on a beach in Los Angeles, California, at sea level; estimate the angle of the resultant vector with respect to the horizontal axis between your location in California and Denver.

a) between 1° and 2°

b) between 0.5° and 0.9°

c) between 0.11° and 0.45°

d) between 0.06° and 0.10°

e) less than 0.05°

35 1.4.2. For a given angle θ, which one of the following is equal to the ratio of sin θ/cos θ?

a) one

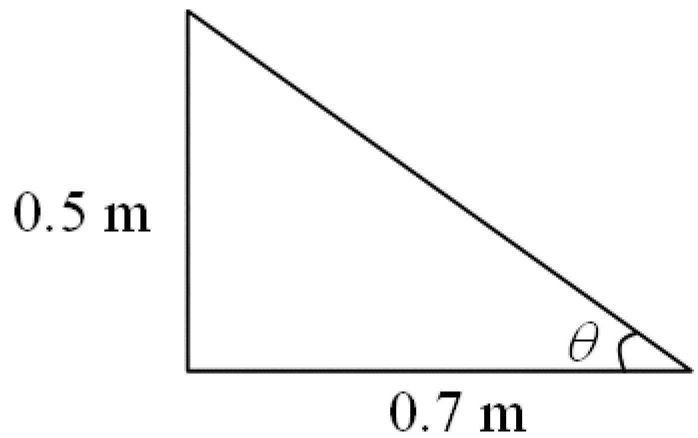
b) zero

c) $\sin^{-1} \theta$

d) arc cos θ

e) tan θ

36 1.4.2. Determine the angle θ in the right triangle shown.



a) 54.5°

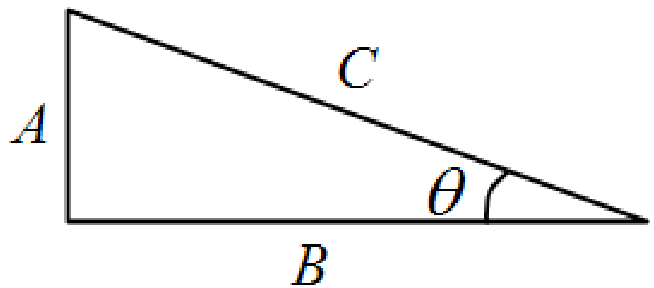
b) 62.0°

c) 35.5°

d) 28.0°

e) 41.3°

37 1.4.3. Referring to the triangle with sides labeled A, B, and C as shown, which of the following ratios is equal to the sine of the angle θ?



a) $\dfrac{A}{B}$

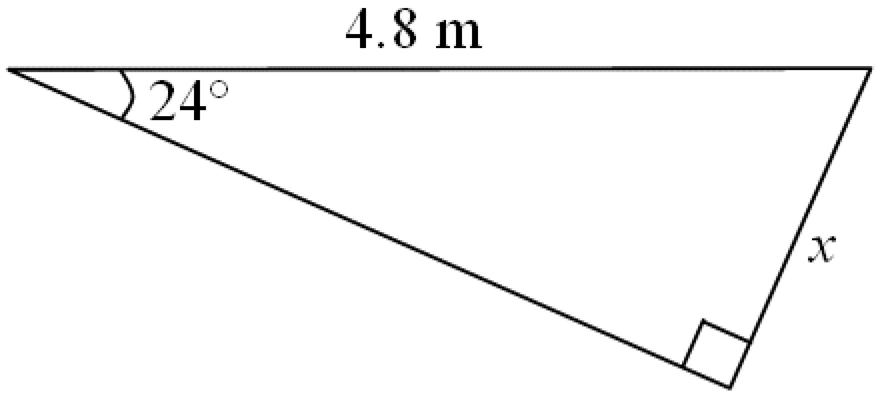
b) $\dfrac{A}{C}$

c) $\dfrac{B}{C}$

d) $\dfrac{B}{A}$

e) $\dfrac{C}{B}$

38 1.4.3. Determine the length of the side of the right triangle labeled x.



a) 2.22 m

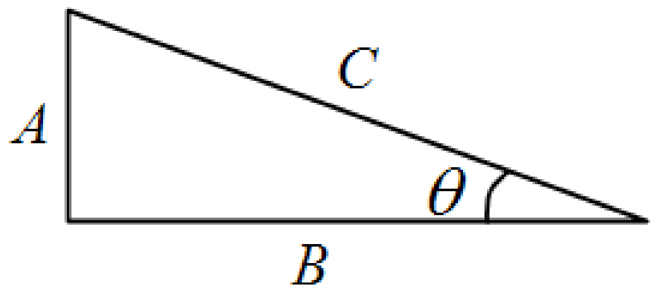
b) 1.73 m

c) 1.80 m

d) 2.14 m

e) 1.95 m

39 1.4.4. Referring to the triangle with sides labeled A, B, and C as shown, which of the following ratios is equal to the tangent of the angle θ



a) $\frac{A}{B}$

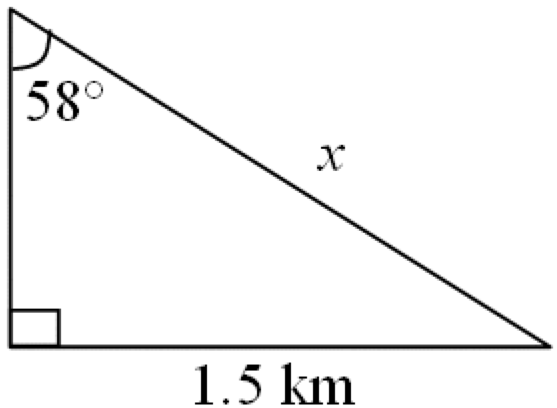
b) $\frac{A}{C}$

c) $\frac{B}{C}$

d) $\frac{B}{A}$

e) $\frac{C}{B}$

40 1.4.4. Determine the length of the side of the right triangle labeled x.



a) 0.79 km

b) 0.93 km

c) 1.51 km

d) 1.77 km

e) 2.83 km

41 1.4.5. Which law, postulate, or theorem states the following: “The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the other two sides.”

a) Snell’s law

b) Pythagorean theorem

c) Square postulate

d) Newton’s first law

e) Triangle theorem

42 1.5.1. Which one of the following statements is true concerning scalar quantities?

a) Scalar quantities have both magnitude and direction.

b) Scalar quantities must be represented by base units.

c) Scalar quantities can be added to vector quantities using rules of trigonometry.

d) Scalar quantities can be added to other scalar quantities using rules of ordinary addition.

e) Scalar quantities can be added to other scalar quantities using rules of trigonometry.

43 1.5.1. Which one of the following situations involves a vector?

a) The submarine followed the coastline for 35 kilometers.

b) The air temperature in Northern Minnesota dropped to –4 °C.

c) The Hubble Telescope orbits 598 km above the surface of the earth.

d) The baseball flew into the dirt near home plate at 44 m/s.

e) The flock of Canadian Geese was spotted flying due south at 5 m/s.

44 1.5.2. Which one of the following quantities is a vector quantity?

a) the age of the pyramids in Egypt

b) the mass of a watermelon

c) the sun's pull on the earth

d) the number of people on board an airplane

e) the temperature of molten lava

45 1.5.2. Which one of the following statements is true concerning scalar quantities?

a) Scalar quantities must be represented by base units.

b) Scalar quantities have both magnitude and direction.

c) Scalar quantities can be added to vector quantities using rules of trigonometry.

d) Scalar quantities can be added to other scalar quantities using rules of trigonometry.

e) Scalar quantities can be added to other scalar quantities using rules of ordinary addition.

46 1.5.3. A vector is represented by an arrow. What is the significance of the length of the arrow?

a) Long arrows represent velocities and short arrows represent forces.

b) The length of the arrow is proportional to the magnitude of the vector.

c) Short arrows represent accelerations and long arrows represent velocities.

d) The length of the arrow indicates its direction.

e) There is no significance to the length of the arrow.

47 1.5.4. Which one of the following situations involves a vector quantity?

a) The mass of the Martian soil probe was 250 kg.

b) The overnight low temperature in Toronto was –4.0 °C.

c) The volume of the soft drink can is 0.360 liters.

d) The velocity of the rocket was 325 m/s, due east.

e) The light took approximately 500 s to travel from the sun to the earth.

1.6.2. When two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are added together, the resultant vector is $\vec{\mathbf{C}}$. Vector $\vec{\mathbf{A}}$ points in the +*x* direction; and vector $\vec{\mathbf{B}}$ points in the $-y$ direction. Which one of the following expressions is false?

a) $\vec{\mathbf{C}} - \vec{\mathbf{A}} = -\vec{\mathbf{B}}$

b) $\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = 0$

c) $C < A + B$

d) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$

e) $C^2 = A^2 + B^2$

49 1.6.2. Which one of the following statements concerning vectors and scalars is false?

a) In calculations, the vector components of a vector may be used in place of the vector itself.

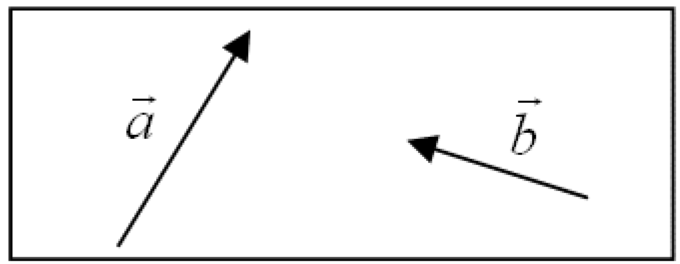
b) It is possible to use vector components that are not perpendicular.

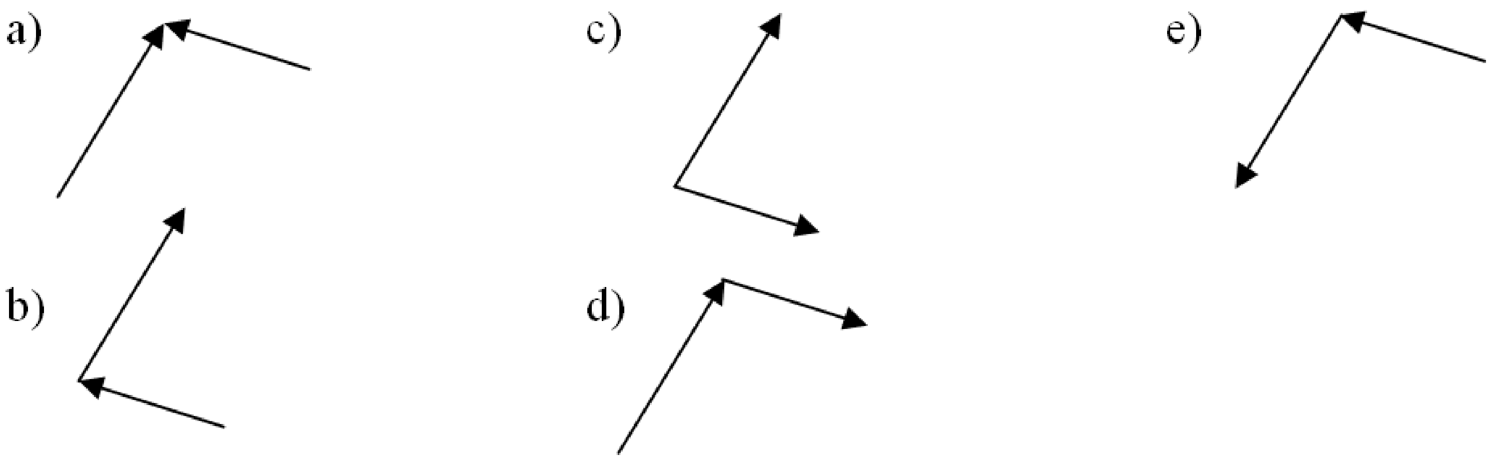
c) A scalar component may be either positive or negative.

d) A vector that is zero may have components other than zero.

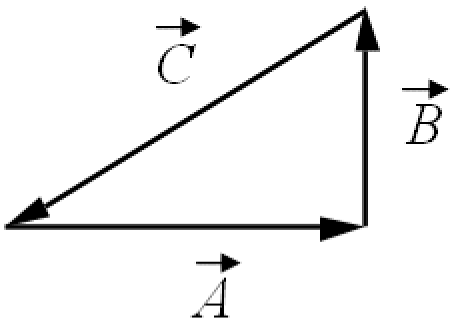
e) Two vectors are equal only if they have the same magnitude and direction.

1.6.3. Consider the two vectors represented in the drawing. Which of the following options is the correct way to add graphically vectors $\vec{a}$ and $\vec{b}$?





51 1.6.3. Which expression is *false* concerning the vectors shown in the sketch?



a) $\vec{C} + \vec{A} = -\vec{B}$

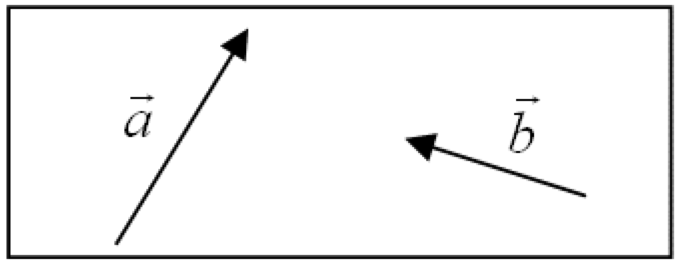
b) $\vec{A} + \vec{B} + \vec{C} = 0$

c) $\vec{C} = \vec{A} + \vec{B}$

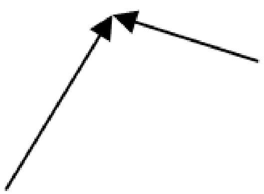
d) $C < A + B$

e) $A^2 + B^2 = C^2$

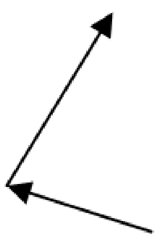
1.6.4. Consider the two vectors represented in the drawing. Which of the following options is the correct way to subtract graphically vectors $\vec{a}$ and $\vec{b}$?



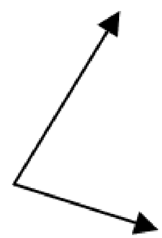
a)



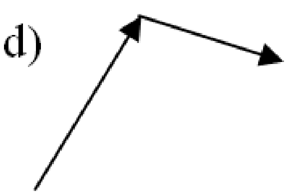
b)



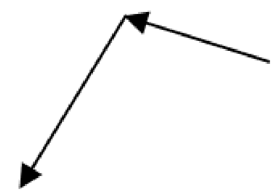
c)



d)



e)



1.7.1. $\vec{A}$, $\vec{B}$, and $\vec{C}$ are three vectors. Vectors $\vec{B}$ and $\vec{C}$ when added together equal the vector A. In mathematical form, $\vec{A} + \vec{B} = \vec{C}$. Which one of the following statements concerning the components of vectors $\vec{B}$ and $\vec{C}$ must be true, if $A_y = 0$?

a) The *y* components of $\vec{B}$ and $\vec{C}$ are both equal to zero.

b) The *y* components of $\vec{B}$ and $\vec{C}$ when added together equal zero.

c) $B_y - C_y = 0$ or $C_y - B_y = 0$

d) Either answer (a) or answer (b) is correct, but never both.

e) Either answer (a) or answer (b) is correct. It is also possible that both are correct.

54 1.7.1. During the execution of a play, a football player carries the ball for a distance of 33 m in the direction 76° north of east. To determine the number of meters gained on the play, find the northward component of the ball's displacement.

a) 8.0 m

b) 16 m

c) 24 m

d) 28 m

e) 32 m

1.7.2. Vector $\vec{r}$ has a magnitude of 88 km/h and is directed at 25° relative to the *x* axis. Which of the following choices indicates the horizontal and vertical components of vector $\vec{r}$?

	r_x	r_y
a)	+22 km/h	+66 km/h
b)	+39 km/h	+79 km/h
c)	+79 km/h	+39 km/h
d)	+66 km/h	+22 km/h
e)	+72 km/h	+48 km/h

1.7.2. Vector $\vec{a}$ has components $a_x = 15.0$ and $a_y = 9.0$. What is the approximate magnitude of vector $\vec{a}$?

a) 12.0

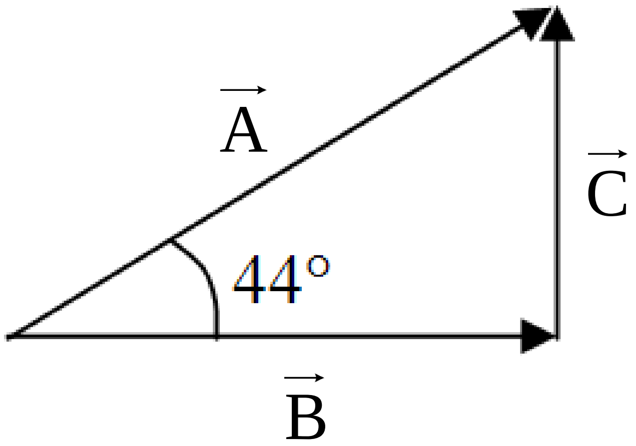
b) 24.0

c) 10.9

d) 6.87

e) 17.5

1.7.3. $\vec{A}$, $\vec{B}$, and $\vec{C}$ are three vectors. Vectors $\vec{B}$ and $\vec{C}$ when added together equal the vector $\vec{A}$. Vector A has a magnitude of 88 units and is directed at an angle of 44° relative to the *x* axis as shown. Find the scalar components of vectors $\vec{B}$ and $\vec{C}$.



	B_x	B_y	C_x	C_y
a)	63	0	0	61
b)	0	61	63	0
c)	63	0	61	0
d)	0	63	0	61
e)	61	0	63	0

1.7.3. Vector $\vec{a}$ has a horizontal component a_x = 15.0 m and makes an angle θ = 38.0° with respect to the positive x direction. What is the magnitude of a_y, the vertical component of vector $\vec{a}$?

a) 4.46 m

b) 4.65 m

c) 5.02 m

d) 7.97 m

e) 14.3 m

1.8.1. Vector $\vec{A}$ has scalar components $A_x = 35$ m/s and $A_y = 15$ m/s. Vector $\vec{B}$ has scalar components $B_x = -22$ m/s and $B_y = 18$ m/s. Determine the scalar components of vector $\vec{C} = \vec{A} - \vec{B}$.

	C_x	C_y
a)	13 m/s	–3 m/s
b)	57 m/s	33 m/s
c)	13 m/s	33 m/s
d)	57 m/s	–3 m/s
e)	–57 m/s	3 m/s

1.8.2. The horizontal and vertical components of vector $\vec{v}$ are $\vec{v}_x$ and $\vec{v}_y$, respectively. Which one of the following statements concerning the sum of the magnitudes of the two component vectors is true?

a) $v_x + v_x = 0$

b) The sum of the magnitudes of the two components is greater than the magnitude of $\vec{v}$.

c) The sum of the magnitudes of the two components is less than the magnitude of $\vec{v}$.

d) The sum of the magnitudes of the two components is equal to the magnitude of $\vec{v}$.

e) The sum of the magnitudes of the two components is less than or equal to magnitude of $\vec{v}$.

1.8.3. The horizontal and vertical components of vector $\vec{V}$ are $\vec{V}_x$ and $\vec{V}_y$, respectively. Which one of the following statements concerning the vector sum of the two component vectors is true?

a) The sum of the magnitudes of the two components is greater than the magnitude of $\vec{V}$.

b) The vector sum of the two components is greater than the magnitude of $\vec{V}$.

c) The vector sum of the two components is less than the magnitude of $\vec{V}$.

d) The vector sum of the two components is equal to the magnitude of $\vec{V}$.

e) The vector sum of the two components is less than or equal to the magnitude of $\vec{V}$.